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BOOK OF ABSTRACTS

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Generalized Impulse Response Function as a Perturbation of a Global Solution to DSGE Models

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In the conventional perturbation approach for solving DSGE models, the dynamics of the deviation of solutions from the steady state after a shock hitting an economy represents an impulse response function (IRF). A method to construct the IRF as a deviation from a deterministic global solution is proposed. The approach detects asymmetric reactions of an economy to shocks in different initial conditions. For example, in an economic downturn a negative shock might affect the economy more severe than in normal economic conditions. The method allows for constructing the IRF for highly nonlinear DSGE models.

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Comparison Between Two Numerical Methods for Solution of 2D Boussinesq Paradigm Equation

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In this paper we evaluate propagating wave solutions to the two dimensional Boussinesq Paradigm Equation

$$u_{tt} - \Delta u - \beta_1 \Delta u_{tt} + \beta_2 \Delta^2 u + \Delta f(u) = 0, \quad (x,y) \in \mathbb{R}^2,\, t \in \mathbb{R}^+, \tag{1}$$

$$u(x,y,0) = u_0(x,y),\, u_t(x,y,0) = u_1(x,y), (x,y) \in \mathbb{R}^2, \tag{2}$$

where $f(u) = \alpha u^2$, $\alpha > 0$, $\beta_1 > 0$, $\beta_2 > 0$ are dispersion parameters, and Δ is the two-dimensional Laplace operator. Two numerical methods are used to obtain solution for equations (**??**)-(**??**): Energy Saving method, which uses conservative finite difference scheme, and Taylor method, which uses Taylor series expansions around the time variable t. Furthermore, for each method the energy and the mass of the solution are calculated. The results, i.e., the solution, energy and mass, from the two methods are compared. The solution and energy are computed over three nested meshes to examine the convergence of both methods. The energy and mass are found at each iteration step. In the case of Taylor method, the energy is calculated using trapezoidal, Simpson's and Boole's rules with $O(h^2+\tau^2)$, $O(h^4+\tau^4)$ and $O(h^6+\tau^6)$ errors, respectively. In the case of Energy Saving method, the energy is computed using trapezoidal rule. The main tool for testing the convergence rate ξ of all examined finite difference schemes and TS expansions is the Runge's Method. The goal is to justify the Taylor series approach by showing that both methods produce similar results in case of $O(h^2+\tau^2)$ approximation order. Furthermore the Taylor method could be used with higher approximation order to produce finer results. The outcome of the comparison is very good. The maximum difference between the two approaches (among calculated solutions using different parameter sets) in L_2 and infinity norms is 0.009981 and 0.004560 respectively.

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Biomathematics Estimation of Parameters of Gas Diffusion in Carbon Nanostructures Using Molecular Dynamics Method

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In this work we estimate parameters of gas transport parameters in organic nano-porous materials, such as kerogens, which is important for understanding the fundamentals of the processes that occur in natural gas production. We use Molecular Dynamics for estimating the numerical parameterization of the processes of transfer of components of natural gas occurring at the nanoscopic level in organic nano-porous materials. We use modified carbon nanotubes and graphene layers as a model nano-porous materials and we use steered molecular dynamic method for estimating the permeability and diffusion coefficients, which can be then used for modeling of gas transport using computational methods for modeling on larger (mesoscopic) scale. The results of this work in perspective can be used in the search for possible ways to optimize the existing models, used modeling of natural gas production processes.

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On the Approximation of Goodwin's Business Cycle Equation by Taylor Series Expansion

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The paper discusses the possibility of describing the properties of solutions Goodwin's business cycle model [1]

$$\varepsilon\frac{dy(t+\theta)}{dt}+sy(t+\theta)=\varphi\left(\frac{dy(t)}{dt}\right)+A(t+\theta), \tag{1}$$

or

$$e^{\theta D}(\varepsilon Dy(t)+sy(t))=\varphi(Dy(t))+A(t+\theta), \quad D=\frac{d}{dt},$$

by replacing Eq. (1) with the ordinary differential equations of finite order [2-6]. The problem is that, due to the nonlinear dependence of $\varphi\left(\frac{dy(t)}{dt}\right)$ the advance differential equation (**??**) has an infinite number of stable periodic solutions (limit cycles) [2]. Therefore, the differential equations must also have two or more stable limit cycles. In [2-6] it was proposed to use the finite sum of the expansion of the exponential operator in the Taylor series

$$e^{\theta D}\approx L_n(\theta D)=1+\theta D+\frac{(\theta D)^2}{2!}+...+\frac{(\theta D)^n}{n!}, \quad n=1,2,... \tag{2}$$

The case $n=1$ was considered in [1] and the corresponding 2nd order ODE describes the long-periodic Goodwin cycle. For existence of two limit cycles the ODE must be at least 4th order, for three cycles – at least 6th order, etc. We show that the ODE obtained by using Eq.(**??**) describes only Goodwin's cycle, but the short-periodic Bothwell's cycles [2] cannot be obtained in this way.

By using 3/3 Padé approximant for exponential operator

$$e^z=\frac{1+\frac{z}{2}+\frac{z^2}{10}+\frac{z^3}{120}}{1-\frac{z}{2}+\frac{z^2}{10}-\frac{z^3}{120}}$$

we find the fourth-order ODE from Eq.(**??**) that has two stable limit cycles: long-periodic and short-periodic.

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Advanced Stochastic Approaches for Option Pricing Based on Sobol Sequence

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Recently Monte Carlo and quasi-Monte Carlo approaches have become a very attractive and necessary computational tools in finance. The field of computational finance is becoming more complicated with increasing number of applications. The pricing of options is a very important in financial markets today and especially difficult when the dimension of the problem goes higher. Monte Carlo and quasi Monte Carlo methods are appropriate for solving multidimensional problems, since their computational complexity increases polynomially, but not exponentially with the dimensionality. A comprehensive experimental study based on scrambling of the Sobol sequence is applied for the first time to evaluate European style options. The Sobol scrambling method is not only one of the best available algorithms for high dimensional integrals but also one of the few possible methods, because in this work we show that the deterministic algorithms need a huge amount of time for the

evaluation of the multidimensional integral, as it was discussed in this paper. The numerical tests show that the method is efficient for multidimensional integration and especially for computing multidimensional integrals of a very high dimension.

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Symmetric n-Homosemiderivation of Rings

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The study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957. J.Bergen [1] introduced the notion of semiderivations of a ring R, which extends the notion of derivation of a ring R, as follows: $d : R \to R$ is a semiderivation of R if there exists a function $g : R \to R$ such that(i) $d(xy) = d(x)g(y) + xd(y) = d(x)y + g(x)d(y)$ for all x; y in R and (ii) $d(g(x)) = g(d(x))$ for all $x \in R$. In 2000, El Sofy [2] defined a homoderivation on R as an additive mapping $h : R \to R$ satisfying $h(xy) = h(x)h(y) + h(x)y + xh(y)$ for all x; $y \in R$ while in 2020, Mehsin Jabel Atteya [3] introduced the definition of (σ, τ)-Homgeneralized derivations as follows: let R be a ring and σ; τ be automorphism mappings of R. An additive mapping $H : R \to R$ is called a $(\sigma; \tau)$-Homogeneralized derivation of R if $H(xy) = H(x)H(y) + H(x)h(y) + h(x)h(y)$, where $h : R \to R$ is $(\sigma; \tau)$-Homoderivation of R for all x; $y \in R$. The main purpose of this paper is to introduce the definition of a symmetric n-Homosemiderivation of rings. This article divided into two sections, in the first section, we emphasize on the definition of a symmetric Homosemiderivation of rings. In the second section, we study a weakly semiprime ideal and a weak zero-divisor for prime rings and semiprime rings. Examples of various results have also been included.

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Nonlinear Inverse Boundary Value Problem for a Second-order Hyperbolic Equation with Nonlocal Condition

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In this paper, a nonlinear inverse boundary value problem for the second-order hyperbolic equation with nonlocal conditions is studied. To investigate the solvability of the original problem, we first consider an auxiliary inverse boundary value problem and prove its equivalence (in a certain sense) to the original problem. Then using the Fourier method and contraction mappings principle, the existence and uniqueness theorem for auxiliary problem is proved. Further, on the basis of the equivalency of these problems the existence and uniqueness theorem for the classical solution of the considered inverse coefficient problem is proved for the smaller value of time.

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Noise-induced Excitement and Mixed-mode Oscillatory Regimes in the Chialvo Model of Neural Activity

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Stochastic variability of oscillatory behavior of the neuron is studied on the basis of the discrete Chialvo model. We consider a parameter zone near the Neimark-Sacker bifurcation where the model exhibits regimes of mono- and bistability with coexistence of equilibrium and quasiperiodic closed invariant curves. A phenomenon of the noise-induced excitement in the zone where the equilibrium is the only attractor of the system was revealed and investigated. To analyze this phenomenon parametrically, we use statistics of interspike intervals. In the bistability zone, stochastically induced mixed-mode oscillatory regimes are described. Transformations of system dynamics from regular to chaotic and back are discussed.

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General Approach to the Tasks of Decoupling of Linear Controlled Systems

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A general technique for decoupling of equations describing the evolution of various automatic control systems was obtained. For this, has been established a correspondence between the problems of splitting equations containing rectangular matrices of coefficients and reducing auxiliary square matrices to block-diagonal form. The same approach has been applied to the decoupling of matrix macroeconomic models. Thus to the new tasks were moved both computational methods and theoretical methods obtained previously. Here (in contrast to the works of other authors), no restrictions were imposed based on the finiteness or compactness of the symmetry group of the system or on the semi-simplicity used in calculating the algebra. We believe that the use of a priori information about the symmetry of the design scheme have already been performed. The established correspondence between the problems of splitting the evolution equations of a controlled system and reducing square matrices to block-diagonal form is useful not only from the computational point of view. The theoretical results are also extending to the problems of splitting the considered systems of equations. It means, that the sequential application of this method, first to the original system of equations, and then to the resulting subsystems, allows one to obtain the maximum possible number of independent subsystems and, at the same time, obtain subsystems of the minimal order. This also implies the uniqueness of the decoupling of equations into subsystems and the possibility of forming a symmetry group corresponding to the decoupling found. The possibility or impossibility of decoupling depends on the structure of the system under study. Therefore, the developed approach can be interpreted as revealing the hidden symmetry of the system, and the approximate decoupling — as revealing a hidden small parameter.

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Li Coefficients on Functions Fields

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In this paper, we study a Li-type criterion for the zeta function associated to the function field. First, we define two types of generalized Li-type coefficients and relate them with the Generalized Riemann Hypothesis.

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Strong Maximum Principle for Viscosity Solutions of Fully Nonlinear Cooperative Elliptic Systems

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In this paper we consider validity of strong maximum principle for viscosity solutions of the following fully nonlinear cooperative elliptic system

$$G^k(x, u_k, Du^k, D^2u^k) + \sum_{i=1}^{N} c_{ki}u^i = 0$$

for $x \in \Omega$ and $k = 1, ...N$. Here Ω is a bounded domain in R^n such that $\partial\Omega$ satisfy interior sphere condition. The principal symbols G^k are supposed degenerate elliptic ones. Furthermore, functions $G^k(x, s, p, q)$ are assumed non-decreasing with respect to s and Lipschitz continuous with respect to p variable. Functions $c_{ki}(x)$ are supposed continuous, $c_{ki}(x) \leq 0$ for $k \neq i$, and $\sum_{i=1}^{n} c_{ki}(x) \geq \lambda > 0$. The validity of strong interior maximum principle for semicontinuous viscosity sub- and supersolutons of the nonlinear system above is shown, as well as the validity of strong boundary maximum principle for the same system.

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Remarks on the Schwarzian Mechanics

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We present mathematical remarks and comments on the 1D conformal mechanics of de Alfaro-Fubini-Furlan, the isotropic oscillator driven by the conformal mode, and the Galajinsky variant of Schwarzian mechanics. We discuss their relation with the Ermakov-Pinney Equation and the Kummer-Schwarz equation as well as their Lie point symmetry groups.

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Application of Network Economic and Mathematical Modeling of Adaptive Investment Control Optimization for Public Catering

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The article is devoted to the development and application of economic and mathematical models for managing investment projects in public catering based on the use of the feedback principle. The purpose of the work is to describe the practical implementation of optimization of investment management processes in public catering using adaptive optimization methods based on network economic and mathematical modeling. As the objective function of the task, the value of the time duration for the execution of the investment project as a whole is considered and it is required to minimize it. A class of admissible strategies for adaptive control of the process of implementing a specific investment project is formed based on the use of network economic and mathematical modeling and the availability of information about the progress of its implementation. On the basis of these strategies, the method of achieving optimal self-adjusting management of the investment project is determined, the optimal execution time and the optimal schedule for the project implementation are determined. The practical significance of the study lies in the implementation of a new optimization network economic and mathematical model with the possibility of adaptive control of a specific investment project to update the concept of a public catering enterprise. This makes it possible to adapt the process of managing the implementation of an investment project for a catering enterprise, taking into account the influence of various disturbances and minimizes possible negative consequences. Further development of this area may be associated with the development of a computer model for optimizing the adaptive control of investment projecting processes for public catering.

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Time Series Modeling and Forecasting of Deposits in Foreign Currency Using CART Ensemble and Bagging

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Time series data from the financial and forex sectors are highly stochastic in nature, implicitly involving a variety of factors. They are usually a combination of linear and nonlinear partial patterns and volatilities. This paper examines the data on foreign currency deposits of Bulgarian citizens in the period from February 2004 to April 2021, accumulated by months from official bank sources. For statistical processing of deposits, the one-dimensional time series was modeled using the new data mining and machine learning method CART Ensemble and Bagging (EBag). The lag variables of the time series and the identified trend, determined by prior application of the Box-Jenkins ARIMA methodology, were used as predictors. Cross-validation was used to avoid possible over-fitting of the models. By varying the number of trees in the ensemble, it was found that the quality of the models self-regulates and begins to deteriorate above a certain limit. This makes it easier to choose the best model. The best EBag models obtained explain the data by over 93%. The models are applied for short-term forecasts for the next 3 months. The comparison of the obtained forecasted values of the deposits with the actual ones shows a very good coincidence and demonstrates the potential of the proposed approach for modeling financial time series.

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Bifurcation Analysis Tools for Nonlinear Complex Dynamical Systems

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Continuation methods are numerical algorithmic procedures for tracing out branches of fixed points/roots to nonlinear equations as one (or more) of the free parameters of the underlying system is varied. In addition to standard continuation techniques such as the sequential and pseudo-arclength continuation, we will present a new and powerful continuation technique called the deflated continuation method (DCM) which tries to find/construct undiscovered/disconnected branches of solutions by eliminating known branches. In this talk we will apply this method to the Nobel-Prize winning area of Bose-Einstein Condensates (2001 Nobel Prize in Physics). Specifically, we will showcase the application of the DCM to the one- and two-component Nonlinear Schrödinger (NLS) equations in two and three spatial dimensions. We will present novel nonlinear steady states that have not been reported before and discuss bifurcations involving such states. A discussion about challenges in such systems will be offered and a summary of open problems will be emphasized.

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Stochastic Functional Expansion for Identifying the Effective Heat Conductivity Coefficient of Polydisperse Suspension

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We consider a random two-phase medium which represents a matrix containing an array of non-overlapping spherical inclusions with random radii. A statistical theory of transport phenomena in the medium is constructed by means of the

functional (Volterra-Wiener) series approach for identifying the effective heat conductivity of a polydisperse spherical suspension. An approximate model based on power-series expansion of the kernels with respect to the volume fraction is developed. The functional series for the temperature is rendered virial in the sense that its truncation after the p-tuple term asymptotically correct to the order γ^p where γ is the mean number of spheres per unit volume – also proportional to the volume fraction. The case $p = 2$ is considered in detail and the needed kernels of the functional series are found to the order γ^2. The truncated Volterra-Wiener expansion is applied consistently to derive the equations for the kernels and their contributions to the overall (effective) modulus are identified. In this way, not only the effective conductivity, but also all needed correlation functions can be expressed in closed form, correct to the said order.

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Convolution Integrals of Christov Functions

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The Christov real-valued functions were introduced as the combination of the Wiener functions. The functions were and are used as a basis system when a spectral method is applied to soliton problems in $L^2(-\infty, +\infty)$. The functions have proven to be a very useful and reliable numerical tool for the investigation of such problems. The number of terms required in a Christov-Galerkin expansion to obtain very good results is quite small in comparison with other basis systems. The efficiency and accuracy of the method can be further improved if the expansion is centered at a point other than the origin. The necessary convolution integrals of the form

$$\int_{-\infty}^{\infty} C_n(x)C_k(x-y)dx, \quad \int_{-\infty}^{\infty} S_n(x)S_k(x-y)dx, \quad \int_{-\infty}^{\infty} S_n(x)C_k(x-y)dx$$

are computed, enabling the expansion of the shifted Christov functions into Christov functions and vice-versa. The accuracy of the expansions is tested numerically

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25 Years of Dissipative Solitons

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In this talk, I will discuss the concept of a dissipative soliton introduced by C. I. Christov and M. G. Velarde in 1995 [1]. In the 25 years since, the subject has blossomed to include many related phenomena, but the focus of my talk will be the core Christov-Velarde concept of "a production-dissipation (input-output) energy balance" mechanism in a nonlinear evolution equation. First, I will survey the conceptual influence that the Christov-Velarde paper has had on the field of nonlinear wave mechanics. Then, I will present new results from my research group on dissipative solitons generated on the interfaces of a ferrofluid droplet by an external magnetic field [2]. I will summarize our derivation of a long-wave equation in a simplified parallel-flow rectangular geometry, which is of the generalized Kuramoto-Sivashinsky type. This new nonlinear evolution equation exhibits the Christov-Velarde production-dissipation mechanism leading to permanent, traveling nonlinear waves. Intriguingly, in this new equation, periodic (rather than strictly localized) nonlinear traveling waves emerge as the dissipative solitons.

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Special Discontinuities in Solutions of Nonlinear Hyperbolic Equations

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Continuous medium models admitting the appearance of solutions in the form of special discontinuities are discussed. Special discontinuities are discontinuities at which conservation laws and an additional condition are fulfilled. In our models, we obtain an additional condition from the requirement for the existence of a discontinuity structure. The discontinuity structure can be described in different ways depending on the adopted model of small-scale processes occurring within the structure. The description of the structure (accounting for various small-scale processes) defines a set of solutions representing the structures of special discontinuities.

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Computational and Experimental Study on Investment Casted Micro-turbopump Impeller

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The present study was focused on studying the investment casting of a small closed impeller designed for a micro-turbopump. Computational and experimental methods were performed consisting in casting and solidification simulation, additive manufacturing and investment casting methods used for the impeller manufacturing, and also non-destructive techniques were implied to evaluate the manufactured models. ProCAST software was used for the impeller's casting and solidification simulation and the results obtained showed that many areas of the part are prone to casting induced defects. An additive manufacturing method was used to manufacture the wax patterns intended for the impeller's investment casting process. Dimensional accuracies of the wax patterns as well as the cast part was assessed by 3D laser scanning. In order to experimentally determine the defects induced by the casting process, a non-destructive testing analysis by X-ray and CT scanning was performed. Based on the results obtained it was concluded that the material jetting method can be applied to manufacture small wax patterns that present a high accuracy and can be used to investment cast complex shaped parts. Even if the experimental results showed a lower degree of casting induced defects present in the cast part compared with the computational simulation results, using such simulation software is still a good way to design the casting technology and to evaluate the critical areas of complex shaped parts.

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Taylor-Couette Flow in Rarefied Gas at Inhomogeneous Cylinder Wall Temperature Distribution

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It is essential to determine the sensitivity and the inertia of the cylindrical Pirani gauge under different conditions when modeling it. For this purpose, it is appropriate to study the Taylor-Couette flow between two stationary concentric cylinders with finite length. In the work is considered the possibilities for instability and self-organization of the flow in the gaseous medium between the cylinders with the wall inhomogeneous temperature profile of the inner cylinder (fiber). These processes can affect the accuracy and sensitivity of the sensor. The heat and the energy transfer in setting on different forms of temperature distributed on the inner cylinder in the form of stationary and non-stationary (running) temperature wave are studied. The realization of such conditions is difficult technically feasible in some cases, but their set in mathematical model gives more opportunities to study the stability and the flow self-organization in the gaseous medium between the cylinders.

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On Some New Results on Mathematical Modelling of the Human Body: A Short Review

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Mathematical modelling is one of the methods for determination of the mass-inertial parameters of the different segments of the body, as well as of the body as a whole. The current article presents a new 20-segmental 3D mathematical model of the human body, generated in a computer environment, allowing the calculation of the mass-inertial characteristics of all body segments. Some data obtained from it for all segments of the body are reported. We also present a short review of some of our results related to i) some additional anthropometric measurements needed for improving the geometrical modelling of the body segments; ii) study of the mass – inertial characteristics in basic body position as selected by NASA; iii) generated model for the upper limb of the human body. The proposed model is shall be helpful in engineering when designing devices aimed to help disabled people. Then, we outline some additional improvements and future developments of the models

under consideration. Let us note that the proposed models are oriented towards applications in medicine (orthopaedics and traumatology), rehabilitation robotics, computer simulations, sports and areas such as ergonomics, simulation of human behavior in space, forensics and more.

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Modification of the Method of Corrective Functions in Control Problems on the Accuracy of Numerical Analysis of Structures

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During the optimal design of dynamic mechanical systems operating in aggressive external media, special attention should be paid to issues related to computational costs and estimates of the results obtained. Metal structures designed to operate in corrosive external media are considered as the object of research. The main computational costs are incurred in calculating the constraint function in the optimization problem (or in solving the problem of predicting the durability of corroding structures). We propose a modification of the method of correction functions based on polynomial approximation of structural elements when solving the strength problem. The approach consists of two stages: function decomposition and uses the regulator of a neuro-fuzzy network to obtain error estimation. The neuro-fuzzy network uses geometric characteristics of the structure, aggressive environment parameters, stresses in the system elements, and third-degree polynomial coefficients to determine the problem solution error. The obtained approximate solution and its error allow obtaining a more exact solution. The use of adaptive network fuzzy inference systems allowed to significantly reduce the computational cost and obtain numerical solutions with a given accuracy. The results obtained during numerical experiments confirm the correctness of the chosen approach.

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Elliptic Functions for Calculation the Propagation Time of a Signal Emitted by a Moving Satellite

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The propagation time of a signal, emitted by a moving along an elliptical orbit satellite from the GPS (or GLONASS) satellite configurations is a very important ingredient of the theory, based on the formalism of the null cone and accounting for the effects of the General Relativity Theory. For the case of satellites,orbiting along a plane elliptic orbit, it has been proved that the propagation time is given by a combination of elliptic integrals of the first, second and third kind. For the more general case of space-distributed elliptic orbits, the propagation time is expressed by higher (fourth) order elliptic integrals, which according to the standard theory can be expressed recurrently by means of lower-order elliptic integrals.

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Efficient Monte Carlo Algorithms for Integral Equations

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In this paper we propose and analyse different stochastic methods for solving a class of integral equations, namely the second kind Fredholm integral equations. We study and compare different possible approaches to compute linear functionals of the integral under consideration. Error balancing of both stochastic and systematic errors has been discussed and applied during the numerical implementation of the algorithms. An almost optimal Monte Carlo algorithm for integral equations in a combination with the idea of balancing of both systematic and stochastic errors is analysed. Conclusions about the applicability and efficiency of the algorithms have

been drawn. Meaningful numerical examples and experiments with experimental and theoretical relative errors are presented. It is shown that the balancing of errors reduce thecomputational complexity if the error is fixed.

Acknowledgements. The work is supported by the Bulgarian National Science Fund under Young Scientists Project KP-06 M32/2-17.12.2019 "Advanced Stochastic and Deterministic Approaches for Large-Scale Problems of Computational Mathematics" and by the National Scientific Program "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES)," contract No DO1-205/23.11.2018, financed by the Ministry of Education and Science in Bulgaria.

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On Some Aspects of the Distance Computer Education at the Technical Universities, Part I

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We discuss the difficulties and the challenges of the distance education. More exactly, we are concerned with the teaching and learning during the laboratory classes. One of the biggest problems is that very often there are licensed products and it is almost impossible to apply them teaching from home. So that, we suggest different free program products etc. In this part, we illustrate our ideas with the least square method and its applications in different faculties.

Key words: Distance education, programming, least square method.

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Necessary and Sufficient Conditions for the Existence of Non-oscillating and Oscillating Solutions of Quasilinear Functional-differential Equations of Neutral Type with Constant Coefficients and Impulse Effect

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This paper is dealing with second order neutral delay impulsive quazi-linear differential equations with constant coefficients and one constant deviation of the argument. Characteristic equations of such equations are established and conditions about the existence of nonoscillatory and oscillatory solutions of appropriate form of pulsatile exponent or pulsatile line are obtained.

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Pragmatic Approach for Forecasting the Expected Quantity of Consumed Heat Energy

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The energy industry is one of the most important branches, having a strong impact on the global economy and people's lives. Energy is the basis of any modern production, directly affecting the development of other industries. In the production of heat energy, often it is necessary to anticipate the demand for this type of goods in order to make optimal decisions for the development and improvement of this sector. Forecasting the expected amount of heat that will be consumed by a residential building can be a demanding task if done manually. The energy consumption of the building can be represented as a data parameter that changes over a period of time. The usual approach to analyse such time-series data in order to make a prediction is via autoregressive models. One solution of the problem is to perform a hyper-parametrer optimization (grid search) and derive the best results according to a selected metric. However, the grid search approach often is a

time-consuming and resource-intensive task requiring additional human interaction for the final assessment of the most suitable parameters. The purpose of this study is to determine whether a simpler approach can be found and a fully automated algorithm capable of producing a sufficient enough forecast can be developed.

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Numerical Modeling of the Seismic Wavefields in Viscoelastic Soils under Pulse Impact

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Present research is devoted to numerical modeling of the seismic wavefields in geological medium, which is considered as pure elastic or viscoelastic. We use Poynting–Thomson rheological scheme to describe viscoelastic properties of the soils. This scheme is also known as standard linear solid model (SLS). We suppose that the ground has a plane-layered structure with homogeneous layers. This assumption allows us to consider a two-dimensional axisymmetric problem. The numerical algorithm is based on two-cyclic splitting method. After the procedure of splitting we obtain systems of equations describing elastic processes along each axis and a separate system that accounts viscous properties. Computations are performed using a software package developed for the numerical solution of axisymmetric problems. The software complex for multiprocessor computing systems of cluster architecture is written in the Fortran language with the usage of MPI library. This technology allows us to obtain high-resolution images of wavefields. On this basis, we fulfilled numerical experiments for soils with different mechanical characteristics under the influence of a pulse impact.

Acknowledgements. This work is supported by the Krasnoyarsk Mathematical Center and financed by the Ministry of Science and Higher Education of the Russian Federation in the framework of the establishment and development of regional Centers for Mathematics Research and Education (Agreement No. 075-02-2020-1631).

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Pseudo-Riemannian Spaces with Semi-reducible Metrics

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Pseudo-Riemannian spaces can be classified depending on whether they permit reduction of the matrix of the metric tensor to a special form. When a matrix of a metric tensor can be divided into blocks, which contain components depending only on the coordinates inner to the given block, then this space is called a reducible. When components of the second block are multiplied by a certain function of coordinates of the first block, then this space is called a semi-reducible. The class of semi-reducible spaces includes Schwarzschild spaces and some other spaces with interesting applications. There are certain algebraic and differential conditions, which are necessary and sufficient in order to define whether a metric tensor of a pseudo-Riemannian space permits a semi-reduction. These conditions are called a tensor characteristic of semi-reducible spaces. Tensor characteristic of semi-reducibility is not of inner character. Its components cannot be defined via a metric tensor or other inner tensor objects. So, an application of a tensor characteristic for semi-reducibility is often hampered by difficulties. We suggest a special form of description of conditions for integrability of differential equation of semi-reducibility characteristic. We introduce a tensor that permits to simplify algebraic conditions of integrability. We study properties of this tensor and demonstrate how it influences the possibility of semi-reduction of pseudo-Riemannian spaces. We have found necessary conditions imposed on tensors and vectors of a semi-reducibility characteristic for different types of pseudo-Riemannian spaces.

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Simulation of Rarefied Gas Flows – Comparison with Experimental Data

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High fidelity modeling and simulations methods need to be anchored to data collected from selected flight tests or experiments to develop robust, accurate and validated supersonic flow-simulation methods to predict the behavior of flow field throughout the wide range flight regimes including highly rarefied gas flows. We have developed a unified flow model for compressible flows, based on the Generalized Hydrodynamic Equations (GHE) by Alexeev (2004), derived from generalized Boltzmann kinetic equation [1]. The model is supposed to account for kinetic effects (intermediate Knudsen number, fluctuations) in the continuum approximation. This model has been explored for simulations of incompressible viscous flows for a wide range of problems and flow parameters, including high Reynolds numbers flows with thin boundary layers, demonstrating good agreement with experimental data [2]. Simulations of compressible supersonic flows is a very challenging problem as such flows can exhibit both continuum and non-continuum flow regimes. Typically, the flow can be continuous to transitional in the near field flow structure, and free molecular flow in the far field. The shock wave (bow shock) is detached from the vehicle at high altitude, and near boundary slip-flow is typical for such regimes. First results for this model has been reported in [3]. In this paper we provide a comparison of simulation results of the model (called RNS, the Regularized Navier-Stokes) with the experimental data for rarefied hypersonic flows [4,6]. Simulations by DSMC method are also provided, the results by the open source SPARTA DSMC code [5]. The Navier-Stokes model results are provided for comparison too.

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Numerical Simulation and Optimization of Compact High Voltage Pulsed Transformers

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Compact devices for generation of high-voltage pulses are in demand in areas such as portable lightning simulation, expendable electron beam and X-ray sources, field medical instrumentation, oil and mineral exploration, as well as advanced laser power supplies, multi-mode high-power microwave sources, and compact radar technologies (see [1],[2],[3]). The pulsed transformer is a critical component of these devices. It increases a voltage pulse from thousand volts to ten or hundred thousand volts, with the advantage of being compact, efficient, and safe.

Modern nanomaterials, such as NanoPerm, or other advanced ferromagnetics, are quite attractive for use in pulsed transformer cores as they reduce a size and weight compared to steel cores. There are numerous challenges in a transformer design for of high energy pulses of short duration in the micro to nanosecond range. A high magnetic inductance generated in the core may lead to a saturation, reducing the magnetic permeability of these materials. High frequencies (a nanosecond pulse width) cause decline of magnetic properties as well. Keep in mind that magnetic properties of these materials are nonlinear, depend on the field strength (a pulse amplitude), and frequency (a pulse width).

Numerical simulation of pulsed transformers is an efficient way to meet all these challenges, and to find the optimal device design that needs to be compact (pocket size), and safe in use. In this paper, we will present the numerical simulation approach (circuits and fields) and optimization of the compact high voltage pulsed transformers, that we have performed for the development and implementations of the compact (mobile phone size) pulsed power source. The developed device KM-1 has the output pulse voltage 70KV, pulse energy 6J, and the required pulse duration of 1us. This high performance device was able to generate $>$5000 pulses using a miniature cell phone battery (3.7V, 2100mAh), without recharging.

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Image Analysis Algorithms for Measuring the Interfacial Roughness in TBCs Systems

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The thermal barrier coatings (TBCs) often fail by spallation close to the bond coat (BC) - top coat (TC) interface due to the stress induced by the growing of thermally grown oxide layer (TGO) on the rough surface of BC as the oxidation time and temperature increase. The evolution of bond coat roughness during exposure at high temperatures is critical for the lifetime of thermal barrier coatings (TBCs) and cannot be assessed by conventional methods after the top coat was deposited by atmospheric plasma spraying (APS). A non-contact method was developed based on the image analysing and scanning electron microscopy (SEM) techniques to evaluate the BC - TC interface roughness at different stages of oxidation at high temperatures. Mathematical algorithms were applied to extract and process the roughness profile from the SEM images acquired on the BC-TC interface and to calculate the roughness parameters Ra, Rp, Rq, and Rz. To determine the accuracy of the developed method a roughness profile obtained on a roughness comparison specimen by using a conventional stylus profilometer was passed through the processing routine and different filters to define the average line for roughness profile and to calculate the roughness parameters. The method validation was demonstrated on plasma-sprayed TBCs with conventional and nanostructured yttria-stabilized zirconia (YSZ) top coats with the same NiCrAlY bond coat deposited on IN 625 substrate. The TBCs specimens were subjected to isothermal oxidation tests at 1100°C for 100, 200, 300, 400, and 600 hours and the roughness parameters Ra, Rp, Rq and Rz from the metal/ceramic interface were calculated. It was found that isothermal oxidation affects the roughness parameters that generally increase with increasing TGO layer and mixed oxides.

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Optimization of Routes in Transport Logistics

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The features of using the dynamic programming method for planning individual travel routes are considered. The ability to build a multi-step decision-making process in the trip optimization problem guarantees prompt route changes in the event of emergency situations. This is ensured by Bellman's principle of optimality, according to which the optimal route contains many sub-routes, each of which, in turn, is optimal. The constructed optimization model and computational dynamic programming scheme are flexible in the sense of the possibility of including various modifications of the problem, for example, the use of additional optimization criteria.

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Organization of the Weekly Working Hours of Vehicle Drivers When Performing Road Transportation for Urban Logistics

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The study examines the organization of freight transport in small batches in the conditions of urban logistics, a brigade organization of driver's work and even distribution of their total weekly working time. A mathematical model of the problem has been developed. The model is a partially integer nonlinear problem of class NP complete problems. To achieve the formulated goal, a criterion is applied that the difference between the driver who worked the most time and that of the driver who worked the least time, to be minimal. The total number of solutions is 2^{m*n*p} (m - number of drivers, n - number of routes, p - number of working days). This number grows extremely fast even at small parameter values. Complete exhaustion of all possibilities is practically impossible because it requires a lot of memory and computational time. For this reason, the classical approaches with complete exhaustion of possibilities are inappropriate. An alternative to these approaches are genetic algorithms. The choice of genetic algorithms is determined by the nonlinearity of the objective function, the discrete nature of some of the variables and especially the dimensionality of the task. The calculations and programs were made using MATLAB version 2017b, using the especially built-in *ga* function of MATLAB (Solver *ga*), based on genetic algorithms. As a result, the technical performance indicators of the vehicles on delivery routes in urban logistics, related to the work of drivers were assessed. The weekly work of the drivers on the routes is planned according to the criterion of minimal difference in the working hours between the driver who worked the most time and the driver who worked the least time on the considered routes.

Keywords: Urban logistics, genetic algorithms, organization of driver's work, transport optimization

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TSA & ML Predictive Modelling of EU Financial and Economic Indices

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In this work an automated mathematical algorithm is proposed for assessment and evaluation of some of the European Union countries' financial and economic indicators. We have considered the harmonized index of consumer prices, business climate indicator, gross domestic product at current market prices and long-term interest rate for convergence assessment purposes. (S)ARIMAX and NARXNN time series analysis tools are applied to model and forecast the future indicator values. The data is gathered from Eurostat (European Statistics Office). A robust and easy-to-implement algorithm for automated prediction is suggested and a detailed explanation about the design of the steps is given. The forecasted values are justified and an interpretation is made for their economic significance. The paper is concluded with important remarks for the future development of the economic and financial environment of the European Union.

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Parameters Reconstruction in Modeling of Honeybee Colonies Infested with Varroa Destructor

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Recently, massive honeybee colony losses were ubiquitously observed and such were reported particularly in Europe and Bulgaria (see [1,2,3]). To study the colonies collapse, we solve a honeybee population dynamics identification problem for the parameters which are not directly observable in the model, derived in [4]. It models the infestation of a honeybee (Apis melifera) colony by the Acute Bee Paralysis Virus (ABPV), which is transmitted by parasitic varroa mites (*Varroa destructor*) as a vector. What is more, we study in detail the cases of the two-dimensional bee-mite submodel and the one-dimensional healthy sub-model. We apply an adjoint equation optimization method to solve the inverse problem. Having these parameters obtained numerically, we perform a qualitative analysis of the model. The paper is concluded with vital remarks and implications concerning the honeybee colony health.

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On the mKdV Equations Related to the Kac-Moody Algebras $A_5^{(1)}$ and $A_5^{(2)}$

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This paper is a natural extension of Refs. [1,2]. First we outline the derivation of the mKdV equations related to the Kac–Moody algebras $A_5^{(1)}$ and $A_5^{(2)}$. First we formulate their Lax representations and provide details how they can be obtained from generic Lax operators related to the algebra $sl(6)$ by applying proper Mikhailov type reduction groups $\mathbb{Z}_h$. Here h is the Coxeter number of the relevant Kac–Moody algebra. Next we adapt Shabat's method for constructing the fundamental analytic solutions of the Lax operators L. Thus we are able to reduce the direct and inverse spectral problems for L to Riemann-Hilbert problems (RHP) on the union of $2h$ rays l_ν. They start from the origin of the complex λ-plane and close equal angles π/h. To each l_ν we associate a subalgebra $\mathfrak{g}_\nu$ which is a direct sum of $sl(2)$–subalgebras. Thus to each regular solution of the RHP we can associate scattering data of L consisting of scattering matrices $T_\nu \in \mathcal{G}_\nu$ and their Gauss decompositions. The main result of the paper is to extract from T_0 and T_1 related to the rays l_0 and l_1 the minimal sets of scattering data $\mathcal{T}_k$, $k = 1, 2$. We prove that each of the minimal sets $\mathcal{T}_1$ and $\mathcal{T}_2$ allows one to reconstruct both the scattering matrices T_ν, $\nu = 0, 1, \dots 2h$ and the corresponding potentials of the Lax operators L. Following [3] we demonstrate that the the mapping from L to $\mathcal{T}_k$ is a generalized Fourier transform.

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Challenges for Modeling Nuclear Structure: Are the Proton and Neutron Masses and A-body Interactions Relevant?

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We discuss some of the challenges that future nuclear modeling may face in order to improve the description of the nuclear structure. One challenge is related to the need for A-body nuclear interactions justified by various contemporary nuclear physics studies. Another challenge is related to the discrepancy in the NNN contact interaction parameters for 3He and 3H that suggests the need for accurate proton and neutron masses in the future precision calculations.

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Study of the Tensile Strength of Alloy Steels Using Polynomial Regression

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The objective of this study is to identify the influence of the chemical composition of alloyed steel on the tensile strength through predictive multiple regression models of the first and second degrees. Data on the percentage content of nine chemical compounds in alloy steel are used as independent variables: C, Cr, Mn, Mo, Ni, P, S, Si, Al, and the product's diameter. In order to accurately perform multivariate linear regression, all variables undergo Yeo-Johnson transformation in advance to achieve normal or near-normal distribution. The obtained linear regression models fit the measured values for tensile strength with 76% and the models with predictors up to second order — with 97.6%. The alloy compounds with the strongest influence on tensile strength are identified.

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Using MARS for Modeling and Predicting Tensile Strength of Low Alloy Steels

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In this study, highly effective statistical models are built based on the measurements of tensile strength of 60 types of low alloy steels with the help of the powerful and flexible data mining method of multivariate adaptive regression splines (MARS). The percentage contents of nine alloyants C, Cr, MN, Mo, Ni, P, S, Si, Al and the diameter of the test sample are used as independent variables. Linear and non-linear MARS models with partial second and third degrees interactions between the predictors are built and studied. The coefficient of determination of the predicted values for tensile strength reached up to 97% and the root mean square error of the prediction set decreased to 47.754 MPa. For the examined sample, the influence of the independent variables on tensile strength was determined with the most significant one being that of nickel, followed by Mn, Mo and P. The methodology developed to study the mechanical properties of steels using an accurate elemental analysis of alloyants by means of MARS is proposed for the first time in the literature. Moreover, it will expand the potential applications of data mining and machine learning techniques such as MARS in intelligent analysis and prediction of the mechanical engineering experiment.

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On Invariants of Yang-Baxter Maps

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We will discuss Yang-Baxter maps related to matrix refactorisation problems and discrete integrability. We will describe invariants of such maps, which are analogues of conserved quantities in Classical mechanics. In addition, we will describe briefly the Poisson structures related to Yang-Baxter maps and their Liouville (or complete) integrability. This generalises the idea of “canonical transformations.”

Along with a brief review of well known discrete maps, we will present recent results about Liouville integrability of integrable maps related to Grassmann-

extended discrete nonlinear Schrödinger and discrete derivative nonlinear Schrödinger equations.

This is a joint work with Sotiris Konstantinou-Rizos.

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Kähler Generalized Ricci Recurrent Spaces

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Applying a pseudo-Riemannian space as a model for a process in the real world, we often encounter obstacles of technical or technological character. There are several ways to deal with these difficulties. We can introduce additional limitations on the inner objects of a pseudo-Riemannian space. Objects are called inner, when they are built with an application of a metric tensor of the space. Ricci tensor is an inner object of a pseudo-Riemannian space. A tensor of a particular type or special conditions imposed on a Ricci tensor are often applied in the deformation theory, general relativity theory and other fields of application of pseudo-Riemannian spaces. A pseudo-Riemannian space is called Ricci generalized recurrent space when a covariant derivative of Ricci tensor can be expressed as a linear combination of Ricci tensor and some gradient vectors. It is proved that there is no a Kähler Ricci generalized recurrent space that differs from a Ricci symmetric space. A Ricci symmetric space is a space whose covariant derivative of Ricci tensor equals zero. It is demonstrated that such spaces are quasi-Einstein spaces by a necessity. We introduce analogous spaces that account for particularities of Kähler spaces. There is a structure applied in a linear combination, which defines a pseudo-Riemannian space as a Кдhler space. We studied certain properties of these spaces, related, in particular to the Bochner's tensor. Bochner's tensor is an analogy for Weyl tensor of conformal curvature in Kähler spaces. The way is found in which algebraic and differential conditions imposed upon Bochenr's tensor define the properties of Ricci tensor. In order to simplify algebraic conditions of differentiating main equations, we introduced a special tensor that is not inner. We define the way in which this tensor's peculiarities influence properties of Kähler space.

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A New Optimization Approach for Sparsity Based Non-linear Reconstruction in Current Density Impedance Tomography

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Current density impedance imaging (CDII) is a non-invasive imaging technique that can measure the conductivity distribution inside a medium. This can help in identification of abnormalities inside a human being, especially tumor that is known to have high conductivity. In this talk, we present a new framework based on a non-linear optimization approach for the sparse reconstruction of log-conductivity in CDII. The new framework comprises of includes minimizing an objective functional involving a least-squares fit of the interior electric field data corresponding to two boundary voltage measurements, where the conductivity and the electric potential are related through an elliptic PDE arising in electrical impedance tomography. Further, the objective functional consists of a L^1 regularization term to incorporate sparsity patterns in the conductivity and a Perona-Malik anisotropic diffusion term to enhance the edges to facilitate high contrast and resolution. The motivation of this framework primarily comes from some similar recent approach to solve an inverse problem in acousto-electric tomography. Several numerical experiments and comparison with an existing method illustrates the effectiveness of the proposed method for superior image reconstructions of a wide-variety of log-conductivity patterns.

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Study of the Spread of the COVID-19 Pandemic by SVEIRS Model

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After the appearance of COVID-19 at the end of 2019, scientists around the world are creating epidemiological mathematical models aimed at studying and predicting the infection. With the advent of vaccines, the emphasis has shifted to create models that the impact of the vaccination process is taken into account. In this paper an epidemiological mathematical model SVEIRS (Susceptible-Vaccinated-Exposed-Infected-Recovered-Susceptible) is described by a system of five nonlinear ordinary differential equations (ODEs). Stability conditions for the presented model are founded. Numerical and stochastic approaches are used to study the dynamics of the system. The reproductive number is also investigated. Numerical experiments are performed and analyzed.

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On Dynamics of the Chain of Spots in Stratified Fluid

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The task of dynamics of the chain of spots of mixed liquid in stratified environment is considered. Initially, the chain of spots is located on a horizontal level with periodic boundary conditions along the horizontal axis. The task is described by the Navier equations – Stokes equations in the Boussinesq approximation. Under the influence of hydrodynamic forces, this chain turns into a strip of the width of $p/2$, if radius of spot is 1. Salinity is chosen as a stratifying component, just as it can be implemented in the laboratory. This task is of interest both to theorists in terms of the theoretical hydrodynamics of stratified liquid, and experimenters, in terms of the time of the establishment of a stationary field. The task is also of interest to the computer scientists in terms of quality check of computational algorithms. The task is described by the Navier-Stokes equations in the Boussinesq approximation. To solve the problem, the SMIF method (Splitting Method for Incompressible Fluid) is used, of course, the finite different scheme of which has such properties as the second order of approximation on spatial variables, minimal scheme dissipation and dispersion, performance in wide range of Reynolds and Froude numbers and, most importantly, when solving wave processes are a property of monotony. The results of the formation of a strip of stratified liquid depending on the numbers of Reynolds and Froude will be presented.

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Numerical Solution of Spectral Space-Fractional Diffusion Problems: Recent Advances and Challenges beyond the Scalar Elliptic Case

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Let us consider the equation $A^{\alpha}u = f$, $0 < \alpha < 1$, where A is a symmetric positive definite operator corresponding to a second order elliptic boundary value problem in a bounded domain $\Omega \in R_d$. We assume that the non-local fractional diffusion operator A^{α} is defined though the spectral decomposition of A. The current advances in numerical methods for such spectral spatial-fractional diffusion problems have been studied in [2]. Basically, the following three equivalent representations of the solution are used in developing such numerical methods: (i) Dunford-Taylor integral formula (or its equivalent); (ii) extension to a second order elliptic problem in $\Omega \times (0, \infty)$ or to a pseudo-parabolic equation in the cylinder $(x, t) \in \Omega \times (0, 1)$; (iii) best uniform rational approximation (BURA) of z^{α} on $[0, 1]$. Though substantially different, each of these methods can be interpreted as rational approximation of $A^{-\alpha}$. In this sense, there are certain advantages of the approach (iii). The first part of the talk contains theoretical analysis of the BURA method that includes in particular the exponential convergence rate with respect to the degree of rational approximation. The error estimate is independent of the spectral condition number of the related discrete operator, see [3]. The BURA algorithm has computational complexity $O(N \log N)$. The presented numerical experiments include representative model problems illustrating the theoretical estimates as well as a truly large-scale problem in realistic 3D geometry and unstructured FEM mesh for a pure Neumann fractional diffusion equation [1]. Some challenges beyond the scalar elliptic case are considered in the last part of the talk. The discussions include the operator splitting schemes for systems of time dependent fractional-in-space diffusion-reaction equations.

Keywords: Fractional diffusion; robust methods; computational complexity.

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Nonlinear Approaches for Joint Modeling of Clinical Trial Data: Convergence, Efficiency and Precision

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Joint Models (JMs) are a class of statistical parametric models bringing together repeated measures and failure time data into a unified framework [1,2]. Unfortunately, most approaches for fitting of JMs for clinical trial data are iterative and have serious issues, either with convergence or computational efficiency or precision. Two powerful nonlinear (iterative) approaches have been developed for rather specific classes of JMs; both having little or no issues with convergence for their specific classes of problems. The Stochastic Approximation Expectation-Maximization (SA-EM) approach [3] has successfully tackled JM problems from Pharmacokinetics (PK) and Pharmacodynamics (PD) [4], whereas the Stochastic Process Model (SPM) approach has tackled problems from biodemography of aging in humans [5,6]. SAEM uses a stochastic approximation procedure to estimate the conditional expectation of the complete-data likelihood, and allows for model selection using a criterion based on a penalized version of the observed-data likelihood (ie, performing model selection when there are values missing at random, MAR). On the other hand, the SPM is based on the biologically motivated assumption of a quadratic (U- or J-shaped) hazard function and also on a high-dimensional random walk for individuals, and exploits the mathematical formalism of stochastic differential equations (SDEs) and the associated Fokker-Planck equation for the population [7]. We studied the performance of the SAEM and the SPM on JMs for clinical trial data, paying special attention to their convergence properties. A simulation study was conducted to compare the SAEM with SPM, with respect to convergence, execution time, bias (%), and confidence intervals coverage.

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Synthetic Likelihood Approximation Inference for Joint Models of Longitudinal and Survival Data with Analytically and Numerically Intractable Likelihood

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Joint Models (JMs) are a class of statistical parametric models bringing together longitudinal (repeated measures) data and time-to-event/survival data into a unified framework [1,2]. A JM regresses an observed dependent vector, y (comprising time-to-event and time-dependent marker variables) on predictor variables. Maximum Likelihood (ML) inference for JMs is a popular approach for statistical inference on the vector of unknown model parameters, θ (for fixed and/or random effects) [3], but the likelihood function $f(y, \theta)$ is often analytically or numerically intractable for JM which makes ML problematic [4]. We consider only generative JMs (i.e., JMs that one can generate data from) and develop approximate methods for their estimation. To that end, we have adapted the Synthetic Likelihood (SL) approach of Wood [5] that was originally designed for approximating the likelihood of chaotic or near-chaotic dynamical systems models. The SL utilizes a vector, s, of model summary statistics (that could be generated and reproduced by the model with input the observed vector, y) to draw statistical inference on the vector of unknown model parameters, θ. The SL gives access to much of the machinery of likelihood-based statistical inference, and could be considered an indirect method for inference based on intermediate statistics, s. For example, using regression coefficients as the summary statistics vector s promotes approximate normality in the distribution of s, supporting the key multivariate normality approximation $s \sim N(\mu\theta, \Sigma\theta)$, conditionally on the parameters θ [6, 7]. The unknown mean vector, $\mu\theta$, and covariance matrix, $\Sigma\theta$, are themselves generally intractable functions of the vector of unknown model parameters, θ, but they can be estimated, for any particular value of θ, by simulation from the (generative JM) model, in which case a synthetic (log-)likelihood function, ls, can be maximized by a Metropolis-Hastings sampling MCMC. We have evaluated empirically the precision of the SL approximation of the parameter estimates, based on relative bias (%), mean square error (MSE), and coverage probability of the confidence intervals, for a JM of a simulated dataset with a limited number of random effects. Finally, the SL for JM approach is illustrated on a real dataset from biomedicine.

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Prediction of Survival Models: A Comparison Between Machine Learning and Cox Regression

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Machine Learning (ML) is a popular approach for modeling large complex data. Survival data (SD) are characterized by censoring, and to avoid significant bias, several well-established ML methods have been adapted for analyzing SD. Additionally, although ML approaches have been shown effective for large datasets, it is unclear how they compare with classical statistical approaches on relatively small data, eg data generated from clinical trials. Here we perform in-depth comparison of three SD-adapted ML methods (XGBoost for survival, Random survival forest, and Survival neural network) with the classical Cox regression, on both large and relatively small (with few predictor variables) datasets. For the large dataset, different model selection approaches for the Cox regression model were studied. The model comparison was carried out on real as well as on simulated survival datasets where the simulated ones mimicked the real datasets. The pairwise model comparison criteria included SHAP (SHapley Additive exPlanations) values [1-4], Concordant index (C-index) [5], and bootstrap cross-validated Brier [5,6] and Kullback-Leibler [7] scores.

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Technological and Economic Justification for the Production of Electricity through Photovoltaic Generators

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In the present work, on the basis of current data on prices, manufacturers of photovoltaic systems and the electricity market, a technological and economic justification for the production of electricity through photovoltaic generators is presented. The main task is to assess the effectiveness and return on such an investment in the context of the green deal and the new energy and environmental policy of the European Union.

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Comparative Analysis of Different Types Cylindrical Inductors for Induction Heating

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In the present work several different types of cylindrical inductors are investigated. Comparison of the following parameters are made – temperature distribution in different loads, electrical losses in each turn of the inductors and current density distribution in the inductors. We look into the advantages and disadvantages in different applications. We also consider in this paper the most common available on the market copper pipes for production of inductors.

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One Method of Trend Analysis of Technical Objects Conditions

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The trend analysis methods have found wide application in assessing the conditions of complex technical objects. The initial data for the analysis of their technical conditions are multidimensional time series of the measured diagnostic parameters in various operating modes. For gas turbine engines such parameters are: turbine speed, turbine exhaust gases temperatures, compressor pressure ratio, fuel consumption, etc. The authors propose an improved method for trend analysis of the technical conditions of gas turbine engines in long-term operation. At the first stage of using the proposed method, the initial data are converted into time series of deviations of the analyzed diagnostic parameters from the reference dependencies (gas turbine engine throttle characteristics), which makes it possible to propose a multi-mode statistical model of data generation. This model makes it possible to establish the deformation of the throttle characteristics during the long-term operation of gas turbine engines. At the second stage, the well-known scalar trend analysis methods (SSA, caterpillar, principal component method, etc.) and the proposed multidimensional trend analysis methods are applied to the obtained multidimensional time series of deviations from the original model to identify trend components and statistically related trends in parameters. At the second stage, it is proposed to apply complex union of time series and singular value decomposition of a beam of complex trajectory matrices. At the third stage, the characteristics of the time series of residual deviations are assessed to establish the validity of the hypothesis that they belong to the sample from the general population of independent normally distributed random variables. The proposed method improves the reliability of statistical conclusions about the technical conditions of the gas turbine engines.

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Analysis of the Temporal Distribution of Passenger Traffic in Road Transport for the Regional Road Network

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The indicators of passenger traffic on road transport for the regional transport network, obtained on the basis of data from online ridesharing services, are considered in the article. The nature of the intra-day distribution of passenger traffic with its division into categories is studied. The features and patterns of daily fluctuations in passenger traffic are also analyzed. Seasonal and cyclical fluctuations in passenger traffic volumes are modeled using harmonic analysis methods. Short-term forecasts and conclusions of the study are made based on the results obtained.

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A Molecular Dynamics Study of Dislocation-Interface Boundary Interactions in Lath Martensite

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This study aims at extending the current understanding of the nano-scale deformation mechanisms in lath martensite. Despite its long history and use, efforts to better understand the mechanical behavior of lath martensite are still ongoing. The microstructure of lath martensite is set by the prior austenite grain size and each grain is divided into packets of martensite laths [1-3]. Each packet comprises blocks and even sub-blocks, and these, in turn, are constructed from parallel laths of martensite. Lath boundaries within a block are low angle, taking up the small misorientation. Block boundaries are high angle twist boundaries and likely to be opaque to slip [1,2].

Understanding and modelling the details of the dislocation-lath (block) interfaces interactions is a key for understanding softening behavior and strengthening mechanisms of lath martensite and is of basic scientific and practical importance. A difference in dislocation transmission across high-angle and low-angle boundaries in martensite was observed by in-situ nanoindentation in a transmission electron microscope (TEM) [4]. At low-angle lath boundaries dislocation motion is hindered by dense arrays of dislocations that pile up against the boundary. As a critical stress is reached a high density of dislocations is emitted on the far side of the grain boundary into the adjacent grain. Surprisingly, at a high-angle block boundary dislocations are simply absorbed into the grain boundary plane with no indication of pileup or the transmission of strain. In the present work, molecular dynamics simulation was employed to reveal the nano-scale mechanisms enabling the observed interface plasticity, which remain mostly unclear so far. The simulations reveal the existence of a network of sessile geometrically necessary interfacial dislocations in the low-angle grain boundary, which increase the blocking strength of the lath interfaces. At a high-angle block boundary, 1/2[111] screw dislocations are absorbed into the grain boundary plane. At stress of 7 GPa simulations reveal the formation of mobile 1/2[111] and [100] interfacial screw dislocations and emission of mobile dislocations in both crystals. Dislocation gliding within the interfacial planes supposedly enables the observed sliding motion of adjacent martensite blocks along the interface.

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Random Forest Regression for Statistical Modeling and Forecasting of PM10

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PM10 (Particulate Matter with diameter of 10 microns or less) is a major air pollutant with a number of harmful effects on human health. As a primary pollutant, it is also an indicator of the overall level of the ecological state of the environment. This determines the need and appropriateness of research on the accumulated empirical data, especially in affected areas. This paper examines the average daily PM10 levels in a specific region of Bulgaria, near the Black Sea coast and a large refinery for oil and petroleum products. The data of PM10 and meteorological variables such as air temperature, humidity, wind speed and others for a period of more than 6 years are studied. Using the Box-Jenkins ARIMA model, an autoregressive term was identified in the PM10 time series, which was used as an additional predictor in the models. Regression analysis with the Random Forest (RF) machine learning method is used for statistical modeling of the time series. RF models were obtained describing the data by 95%. The models are applied for short-term forecasts of PM10 pollution with 3-5 days ahead. The comparison with the actual measurements showed that the proposed approach gives very good results and could be embedded in mobile software for air pollution forecasting.

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Multicomponent Fokas-Lenells Equations

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We formulate multi-component integrable generalizations of the Fokas-Lenells equation [1,2] which are associated with each irreducible Hermitian symmetric space. We provide a description of the underlying structures associated to the integrability, such as Lax formulation and the bi- Hamiltonian formulation of the equations. Two reductions are considered as well one of which leads to a nonlocal integrable model. Two examples with symmetric spaces of types A.III and BD.I are presented in details.

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Merging of Bivariate Compound Binomial Processes with Shocks

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The paper investigates a discrete time binomial risk model. The last allows different types of polices and shock events may influence some of the claim sizes. It is shown that this model can be considered as a particular case of the compound binomial model. As far as we work with parallel binomial counting processes in infinite time if we consider them as independent the probability of the event they to have at least once simultaneous jumps would be equal to one. We overcome this problem by using thinning instead of convolution operation. The bivariate claim counting processes are expressed in two different ways. The characteristics of the total claim amount processes are derived. The risk reserve process and the probabilities of ruin are discussed.

The results are applied in some particular cases.

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The Predictor-Corrector Mode for the Symmetric Multi-Step Methods for the Numerical Modeling of Satellite Motion

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The orbital motion is described by the system of second-order ordinary differential equations which numerical integration by the Stormer-Cowell multistep methods leads to a longitude error which increases quadratically in time. This presents a problem when performing long-time integration. J. Lambert and I. Watson proposed the symmetric methods, that possess a periodicity property when the product of the step-size and the angular frequency lies within a certain interval called the interval of periodicity. The numerical integration of orbit by the symmetric methods with the step-size from the interval of periodicity gives the longitude error which increases linearly, whereas the energy error remains roughly constant. The symmetric methods are not uniquely determined even if their order and explicitness are specified. We construct and investigate the high-order symmetric explicit and implicit methods in the "Predict-Evaluate-Correct-Evaluate" (PECE) mode. The methods were selected according to size of the stability interval $\mathrm{P(EC)^{kE}}$ mode, the value of an error constant, behavior of the roots of the stability polynomial, and accuracy of numerical solution of tests problem including simulation of disturbed orbital motion.

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Expansions in Papkovich–Fadle Eigenfunctions in the Polar Coordinate System

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The paper presents some basic points of the theory of expansions in Papkovich–Fadle eigenfunctions in the polar coordinate system. Formulas for the Papkovich–Fadle eigenfunctions corresponding to the boundary value problem of the theory of elasticity for a truncated wedge with free long sides are given. Equations for determining the functions biorthogonal to the Papkovich–Fadle functions are constructed. Examples of expansions into Lagrange series, which are the basis for solving boundary value problems, are given.

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Optimization of Services for Cargo Owners on the Railway Transport Using E-technologies

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Formalization of estimated indicators system for servicing freight owners in railway transport using e-technologies is considered. Methods for assessing service levels for such systems have been developed.

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On the Stationary Model of Universe

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The reason for including a cosmological constant in equation of gravitational field of Einstein-Hilbert was Albert Einstein's decision to construct a static model of Universe. The direct substitution of the pseudo-Riemannian space's metric, which is not dependent on time, into field equation leads to contradiction. The field equation with cosmological constant has a static solution with a closed space section. The equation with cosmological constant also has a solution, which defines a stationary (nonstatic) model of Universe. This solution is an equidistant pseudo-Riemannian space. An equidistant pseudo-Riemannian space permits concircular vector fields. This paper studies a number of fields, which are permitted by a given space. It is also demonstrated that distribution of this number has a lacunar character. We defined a tensor characteristic for spaces, which permit a maximum quantity of concircular fields and are different from spaces of constant curvature. Tensor characteristic, necessary and sufficient conditions are algebraic conditions imposed on Riemann tensor of a pseudo-Riemannian space and differential conditions imposed on some other vectors. The paper classifies the spaces under study into three classes. We characterize the properties for every class. The research is done on spaces with more than 2 dimensions. We obtained equations for a scale factor for every type of spaces of upper lacuna of 4 dimensions. This system of equations includes equations of energy, motion and continuity.

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Stability of Combined Steady Convective Flow in a Vertical Fluid Layer with Permeable Boundaries

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Consider a plane vertical channel bounded by two parallel vertical planes and filled with viscous incompressible fluid. Steady convective flow is generated in the channel due to combined effect of internal heat generation and temperature difference between the planes. Heat is released in the fluid as a result of a chemical reaction. There is also a steady flow with constant velocity through permeable channel walls in the direction perpendicular to the main flow. Steady convective flow in the channel is described by a nonlinear boundary value problem. For some values of the parameters the boundary value problem has two solutions. The solution with the smallest norm is selected as the base flow for the linear stability analysis. Base flow velocity and temperature profiles are found numerically. Linearizing the system of equations of thermal convection in the Boussinesq approximation around the base flow and using the method of normal modes we obtain an eigenvalue problem for a system of ordinary differential equations. Collocation method based on the Chebyshev polynomials is used to discretize the problem. Effect of cross-flow on the stability of the combined flow in the vertical direction on the stability boundary is investigated numerically. Depending on the values of the parameters of the problem there exist regions of stabilization and destabilization of the base flow in the parameter space.

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Finite-Difference Approximation of Mean Field Problem with Limited Management Resource

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In this work, we propose the computational algorithm for the formulation of Mean Field Game problem with the limited management resource during control realization. The differential statement in the framework of Mean Field Game approach is usually transformed from the problem of conditional minimization of a value functional with a condition in the form of the Fokker-Planck-Kolmogorov equation to a system of two Fokker-Planck-Kolmogorov and Hamilton-Jacobi-Bellman equations of parabolic type with an algebraic equation connecting them for choosing a strategy and with the condition of complementary slackness. The approximation of these differential equations is carried out using the Eulerian–Lagrangian technique when the elliptic part is approximated in the Eulerian variables and the transfer operator is did in the Lagrangian ones. This provides a significant improvement in the properties of discrete operators, including their conjugacy. The latter provides an exact transition from the problem of conditional minimization to the method of Lagrange multipliers at the algebraic level. The combination of these techniques allowed us to create an iterative numerical method with monotonous minimization of the value functional at each stage of verifying the constraint of the management resource.

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The Pólya-Aeppli Risk Model with Stochastic Premium Process and Ruin Probability

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In this paper we define a risk model with stochastic premium process, such that the counting processes are Pólya-Aeppli processes [1,2,3]. We derive equations for the non-ruin probability in infinite and finite time and analyze the case of exponentially distributed claims. Martingale approximation of the ruin probability is given.

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Concircular Mappings Preserving a Curvature of Multi-dimensional Platforms

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The generalization for the concept of a Riemannian curvature of a space V_n is a curvature mixed by two directions, which was introduced by E. Cartan. Two-dimensional flat directions were defined by two simple bi-vectors. He introduced formulae for curvatures of three- and four-dimensional flat directions. The further generalization of "curvature" concept was suggested by A.Z. Petrov. He rejected a limitation on simplicity of bi-vectors. Instead, A.Z. Petrov introduced a notion of quadratic curvature and proved that V_4 of a constant quadratic curvature is an Einstein space. S.I. Fedischenko treated the concept of curvature of a pseudo-Riemannian space in relation to m-dimensional platform. The curvature of a space V_n in a given point for a given m-dimensional platform is a scalar curvature of m-dimensional surface, which is geodesic in the given point of the space V_n and is tangent to the m-dimensional platform in the given point, while the m-dimensional platform is drawn on a set of m linearly independent vectors. When a conformal mapping of a pseudo-Riemannian space V_n preserves geodesic circles, then it is called a concircular mapping. Geodesic circle is a curve with a constant first curvature, while the second curvature equals identically to zero. Such special conformal mappings were studied by K. Yano. We have studied concircular mappings, which preserve a curvature of m-dimensional platforms. We have found tensor characteristics, which were independent from the selection of a coordinate system, for these spaces. We have built examples of four-dimensional spaces, which permit such mappings.

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Dynamical Systems Induced by Reaction Networks with Application to Epidemiological Outbreaks

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We discuss several familiar dynamical systems induced by reaction networks used for the modeling and simulation of epidemiological outbreaks [1,2,3]. We are especially interested in dynamical systems that are generated by reaction networks including specific basic reactions such as exponential radioactive decay, logistic or Gompertz growth, etc. We mathematically analyze the solutions to these dynamical systems, visualize them and seek typical shapes in their graphs such as inflection points, asymptotes, absolute and logarithmic rates of change in certain intervals, etc. We then look for relations between these peculiarities of the graphs and the shapes of measurement data sets coming from epidemiological outbreaks with different infection transmission patterns, such as "one-to-one" pattern, endemic environmental disease, etc.

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Infinitesimal Deformations of Metrics of Pseudo-Riemannian Spaces

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Difference of metric tensors of two pseudo-Riemannian spaces is called their deformation. While calculating other inner geometric objects, there is often a need to discard certain parameters. This way leads to the research on infinitesimal deformations of a metric. In this sense, infinitesimal parameters are parameters, which can be discarded not affecting the completeness of the problem under study. Since Saint-Venant's times, the deformation research is reduced to analysis of a system of differential equations. Saint-Venant's equations are the main tool for research on infinitesimal deformations. Saint-Venant's equations are understood here as a set of equations defining the deformation tensor in such a way that the space remains an Euclidean space. Generalized Saint-Venant's equations are conditions under which Riemann tensor is preserved under infinitesimal deformations of a metric tensor of a pseudo-Riemannian space. They are differential equations in covariant derivatives in respect to tensors of Ricci and Riemann. Conditions, which are imposed on tensors used for research on infinitesimal deformations, are both algebraic and differential. Having carried out the research of this type we are able to answer the question: whether the Saint-Venant's equations are true under the pre-defined conditions. The research is carried out locally in tensor form, without limitations on a sign of a metric tensor.

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LMWH as a Prospective IL-6 Blocker: a Computational Study

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Interleukine 6 (IL-6) is a signalling molecule, that has both pro- and anti-inflammatory functions and plays an important role in inflammation, immune response, and hematopoiesis. IL-6 signalling requires a two-stage process – firstly, the cytokine binds to the IL-6 receptor (IL-6Rα) and then a second membrane protein, glycoprotein 130 (gp130) is recruited to form a triple complex, which subsequently dimerises. Elevated IL-6 levels are associated with chronic inflammation and autoimmunity. Moreover, the cytokine was found to play a key role in the development of an acute severe systemic inflammatory response – the acute cytokine release syndrome (CRS), known also as cytokine storm (CS). Hence, inhibition of the IL-6 signalling has great potential in the management and treatment of CS.

In this work we study the interaction of IL-6 with a potential IL-6 inhibitor – low-molecular-weight heparin (LMWH). LMWH are highly positively charged polysaccharide chains. In particular, we performed computer simulations of IL-6, and IL-6–IL-6Rα with a particular hexasaccharide that models a general LMWH chain. Our results reveal the mechanism of heparin inhibitory action on the IL-6 activity.

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Metadynamics Approach to Modeling Peptide-membrane Interactions – A Proof-of-concept Study

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Antimicrobial peptides (AMPs) are considered a promising alternative to traditional antibiotics in diseases caused by multidrug-resistant bacterial strains. AMPs' antibacterial activity is associated with their cationic and amphiphilic nature, considered of importance for their interaction with the negatively charged bacterial membrane. The peptide-membrane systems contains about 400000 atoms for the smallest possible AMPs and can reach a few millions of atoms for the longer ones. This makes the conformation space very large and its investigation requires application of enhanced sampling techniques. Metadynamics is one such technique, which provides information about the multidimensional free-energy surface of the investigated system in terms of a reduced number of variables. There is no specific prescription for the selection of these variables which poses an additional complication, especially for larger or conformationally volatile systems. We present the results of a *proof-of-concept* study aiming at the development of a metadynamics-based protocol for investigation and analysis of AMP-membrane complexes. The model membrane is asymmetric, containing neutral and negatively charged phospholipids, POPE and POPG, in ratios 85/15 and 70/30 in the external, resp. internal layer, thus resembling the *E. Coli* membrane. Two different sets of collective variables are employed for optimisation of the protocol.

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Modeling Data of Covid 19 Pandemic from Different Countries with an Extended SIR Model

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In this paper, we apply an extended SIR model to analyze the data generated by the Covid 19 pandemic. We choose several countries from different continents, estimate the parameters employed in the model and examine the data.

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Mathematical Model of the Thermal Process in an Infinite Cylinder Heated by a Moving Heat Source

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3D printing Technologies is widely used in various industries, such as medicine, engineering, aircraft, rocketry and others. All 3D printing processes use filaments, which act as the building block, to construct parts from the ground up. FDM (Fused Deposition Modeling) is an additive manufacturing method that creates parts additively using a filament. The process deposits the filament layer by layer, according to your 3D model. In this paper, we consider a mathematical model of the temperature field of a filament having the shape of an infinite cylinder, heated by a moving, dispersed on a finite segment (in an extruder) a heat source. The mathematical model is constructed in the form of a nonlinear boundary value problem for the equation of thermal conductivity with conditions of conjugation in three combined areas. By transformations, the problem is reduced to solving the one-dimensional boundary value problems for ordinary differential equations of 2nd order with conjugation conditions on boundary determination equations. The heat source, internal or external, is recorded using a Heaviside step function. In the linear case, the solution of the problem is obtained and a graph of the temperature distribution is constructed. The nonlinear problem is solved numerically. Temperature distributions of polymer thread and metal wire both in the extruder and outside it for different conditions of additive printing process control of details are constructed. Qualitative analysis of problem solutions allows us to conclude

about the temperature distribution of both the filament and the wire, which are heated by dispersed internal or external heat sources. The mathematical model can be used in the development of control systems for the production of parts on a 3-D printer.

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Invariant Transformations Preserving Mappings

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A bijection between points of spaces of affine connectivity is called a mapping. In the common system of coordinates, the difference between objects of connectivity of spaces, which are mapped, represents a deformation of connectivity objects during the mapping. A transformation of two spaces, which are connected by a mapping, into another pair is called an invariant transformation, when it preserves a deformation tensor. During invariant transformation, a sequence of pairs can either lead to the same original two spaces after several steps or be infinite. The sum of objects of connectivity with some positive coefficients is called a shortened mapping. The shortened mapping permits to find additional geometric characteristics of the spaces, which are transformed invariantly. These constructions are instrumental while modeling the objects with similar characteristics. The application was demonstrated thanks to examples of geodesic and conformal mappings of pseudo-Riemannian spaces. The first pair of spaces was selected in order to choose the spaces with high mobility in respect to geodesic mappings. We demonstrated the way in which a mobility changes in course of an invariant mapping and the way it is reflected in the shortened mappings and corresponding connectivities. We studied algebraic and differential conditions, which are imposed on Ricci tensors of spaces, which conform to invariant transformations with preservation of mappings.

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Gravity Models for Regional Public Transport

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Various gravity models are used to analyze and predict the spatial distribution of passenger traffic. Such models allow us to calculate the origin-destination matrix. In this paper, we use ticket sales data to calibrate gravity models for public transport in the Sverdlovsk Region. The resulting models are used to predict passenger traffic.

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A Mixed Boundary Value Problem of the Theory of Elasticity for a Half-strip

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The paper deals with a periodic boundary value problem of the theory of elasticity for a half-strip with mixed boundary conditions at its end. The boundary conditions on the long sides correspond to the periodic continuation of the solution into a half-plane, i.e., the solution is represented in the form of trigonometric Fourier series. The constructed exact solution to the problem is based on using conjugate trigonometric series.

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Splitting Schemes for Incompressible Fluid-structure Interaction Problems

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The presentation will be focussed on a class of recently developed splitting schemes for the Navier-Stokes and linear elasticity equations. They are based on a novel approach that reformulates the equations in terms of a stress variable. It was developed in a recent paper together with P. Vabishchevich (Russian Academy of Sciences). The main advantage of such an approach is that the fluid and the structure equations, when written in terms of a stress variable, become very similar. In particular, it is much easier to impose the boundary condition at the interface. Although at first glance the resulting tensorial problem is more difficult, if it is combined with a proper splitting, it yields locally one-dimensional schemes with attractive properties, that are very competitive to the the most widely used schemes for the formulation in primitive variables. Several schemes for discretization of this formulation will be presented together with their stability analysis.

Finally, numerical results for a several benchmark problem will be presented.

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Characterization of the I-Binomial Process

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In this paper we consider the I-Binomial process. We define the process as a compound Binomial with geometric compounding distribution, and as a discrete time birth process. We show that the definitions are equivalent. Then, using the definitions, we give two characterizations of the process. Some properties and possible applications are given.

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Parallel Implementation of Coupled-cluster Equations for Nucleonic Matter

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We present benchmark results from a parallel algorithm of solving the coupled cluster (CC) equations with singles and doubles by using efficient multiplication using the tensor cores technology integrated in modern GPUs. We compare our results against the Lipkin-Meshkov-Glick model's thermodynamic limit as well as against the ones obtained for infinite neutron matter by G. Hagen [1]. We also reason about the possibilities this algorithm provides for making some headway in incorporating A-body forces into the CC equations.

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Nonlinear Waves in Viscoelastic Maxwell Continuum

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Nonstationary motions of incompressible viscoelastic Maxwell continuum with a constant relaxation time are considered. Equations for the velocity, pressure, and stress tensor form a closed system of first-order partial differential equations. Characteristics of the system of equations with the upper and lower convective derivatives are found. The presence of both real and complex characteristics complicates the formulation of an initial-boundary-value problem. Uniqueness of the initial boundary-value problem of the linearized system in the vicinity of the state at rest

is established. The velocity of propagation of transverse waves was described. It is shown that the decrement of decaying of their amplitude is proportional to inverse of the relaxation time [1]. Two-dimensional unsteady stagnation-point flows of an incompressible viscoelastic fluid are studied theoretically assuming that the fluid obeys the upper convected Maxwell model. For achieving better understanding of the main properties of the governing equations, the system of non-linear equations is transformed to Lagrangian variables. As a result, a closed system of equations of the mixed elliptic-hyperbolic type is obtained. These equations are decomposed into a hyperbolic submodel and a quadrature. The hyperbolic part is responsible for the transport of nonlinear transverse waves in an incompressible Maxwell medium. The system of equations guarantees the existence of the energy integral, which allows one to analyze discontinuous solutions to these equations. It is demonstrated that solutions with strong discontinuities are impossible, though a solution with weak discontinuities can exist. Several numerical examples of the problems of practical interest show that perturbations induced by weak discontinuities in the initial data propagate with a finite speed, which confirms the hyperbolic character of the system [2].

For the model with Johnson–Segalman convected derivative and special linear dependence of the vertical component of velocity, the general solutions were derived. It is remarkable that transition to Lagrangian coordinates enables us to find analytical two-dimensional unsteady solutions of the problem of flow near free stagnation point of UCM media. Even more surprising and striking is the fact that the general solution of the nonstationary problem for a special linear dependence of the velocity component can be found for the case of upper, lower and corotational (Jaumann) convective derivatives [3]. Analysis of the analytical unstationary solution found earlier in [3] provides a new class of stationary solutions [4].

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Search for Scientific Information Through the Use of Machine Learning

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The advantages of using machine learning in search are that the search engine can learn and thus lead to more personalized answers, rather than the most popular results. In well-known search engines, such algorithms have been used for a long time and are constantly being improved, but they are focused on the average user and take into account the commercial component. Finding scientific information, given this, is complicated. For machine learning in search engines, it is necessary to have a history of user actions that have many variables such as: geolocation, date and time, device type, personalization data, keywords and more. Also a necessary component is to understand the query context and motivation, to understand what the user means. Given this, for the system of distance learning, which is relevant in a pandemic, developed intelligent search of scientific material depending on the interests of the user. The search algorithm forms a group of relevant interests depending on previous search queries and the history of open tabs and forms the search result in scientific journals. Each article contains keywords, so the interest group is formed from the keywords of the articles that interest the user. To check the relevance of user interest groups, the algorithm checks the current search queries for compliance with past interest groups, if the user's interests have changed, forms a new interest group for the user. Forms its knowledge base (article link and short description) depending on what information from the search result was useful to the user, and displays these results to other users on a similar search query. To solve these problems, the existing database of the remote system was expanded. The following six tables have been created: to post the history of search queries; for unique search queries (no duplicates); to store keywords; to associate keywords with a unique search query; to link the user to keywords; to store useful user links. An algorithm that works on the principles of machine learning has been created to store keywords and associate keywords with the search query and with the user. The developed intellectual search gave teachers the opportunity in the distance learning system to search only for the necessary scientific information without unnecessary information noise.

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Method of Reducing Equality to Identities in Problems of Economics

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The method for reducing equality to identities is based on the definition of a periodic function of several variables. The solutions or conditions for the existence of a formal proposition are found by reduction to identity. The solution is determined in the form of comparisons modulo the period of returning to the original equality unconditionally true. The original equality is a confirming example and must be determined before starting the search for a solution. The original equality value is determined by simple fit. The study consists in proving the necessity and sufficiency of the solution obtained. The proof of necessity consists in obtaining an identity. The proof of sufficiency consists in obtaining an expression, substituting which, the identity is confirmed.

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Mathematical Model for Studying Volumetric Losses of a Three-Rotor Hydraulic Machine with Bilateral Lantern Meshing Taking into Account Internal and External Leakages

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In the current work mathematical model of criterion equation for studying the volumetric losses of a new type of pump with bilateral lantern is achieved. This model could be in use for summarizing experimental data and to achieve empirical dependence for the volumetric efficiency for this type of pumps. The empirical relation takes in account the part of the internal and the external leakages and the kind of influence of the basic geometric sizes, mode parameters and the fluid's properties. Such a dependence will give possibility for numerical research to determine the optimal relations between the geometric sizes and the parameters of the pump to achieve high performance, while the efficiency keeps relatively high. For this purpose, the processes ongoing in the pump are considered and

eleven parameters, which affect the most to the volumetric efficiency are determine. They include two regime parameters, two parameters, which report the fluid's properties and seven geometric constructive variables of the pump. Using the theory of similitude and dimensionalities analysis, with these variables, the summary model of the criterion equation of the volumetric efficiency with nine criteria of similitude – two complexes and seven geometric simplexes are derived. Starting from the criterial dependence in power form for flue of pipes and channels, the mathematical model, which is valid for this class of pumps is finally derived. The geometric simplexes are joint in one common geometric coefficient.

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Band Preconditioners for Non-Symmetric Real Toeplitz Systems with Unknown Generating Function

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In this paper we study a preconditioning technique for non-symmetric, real Toeplitz systems with unknown generating function. The preconditioner is a band Toeplitz matrix, generated by a trigonometric polynomial. Our aim is to estimate the appropriate trigonometric polynomial that approximates, as well as possible, the unknown generating function f. First, we try to avoid the ill-conditioning of the system $T_n x = b$, by eliminating possible roots of f. From the entries of the coefficient matrix T_n we estimate the unknown function, forming its Fourier expansion, on an equally spaced grid G_n in $(-\pi, \pi)$. Then, we propose a procedure to estimate possible roots of the generating function and their multiplicities, in order to form the trigonometric polynomial that eliminates the roots. An error analysis is presented, proving the efficiency of the preconditioner, if the roots were not estimated accurately. After eliminating the roots, we apply the well-known Remez algorithm for further approximation. An algorithm describing step-by-step this procedure is presented. Suitable numerical examples are demonstrated to show the validity and efficiency of the proposed preconditioning technique, using the Preconditioned Generalized Minimal Residual method (PGMRES).

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Optimal Control Strategies Applied to a Compartmental Epidemiological Model of Dengue Fever Transmission in Brazil

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The relationship among epidemiology, mathematical modeling and computational tools has been allowed constructing and testing of theories about the development and combat of a disease. This work is motivated by the study of epidemiological models applied to infectious diseases, giving particular relevance to dengue fever. The mathematical models studied and tested in this work are composed of mutually exclusive compartments and are based on ordinary differential equations that describe the dynamics of spreaded disease, through the interaction between humans and mosquitoes. An qualitative study is made of them in relation to the balance points and their stability. The spread of dengue fever can be mitigated by means of control measures in the mosquito population, such as the use of insecticides and educational campaigns. In addition, the vaccination process acting as extra protection for the human population control. In this case, the optimal control theory is used to define the necessary conditions to minimize the costs of vaccination and treatment of infected individuals. Both control strategies aim to reduce and eradicate the disease in the population. The presented models are solved numerically in order to obtain the numerical solutions to compare the control strategies applied in the epidemiological mathematical models that describe the dengue fever in Brazil.

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Fuzzy Recognition of Proteins in 2D Electrophoresis in Population Genetics

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The development of the previously created technique of fuzzy recognition of proteins in population genetics during gel electrophoresis for the case of 2D electrophoresis is proposed. For a more complete separation of proteins with a similar mass in a mixture, they are first subjected to isoelectric focusing in one direction, and then electrophoresis is carried out in the perpendicular direction. Isoelectric focusing is a type of zone electrophoresis that separates protein molecules based on the difference in their isoelectric points. Localization of proteins after isoelectric focusing is uneven; in addition, the movement of proteins during electrophoresis occurs no longer along a straight cuvette, but in a plane. All this introduces additional distortions in the numerical determination of the mass of proteins. If several proteins have a similar mass, their traces on the electropherogram intersect. Visually definable boundaries of the carrier of a fuzzy membership function are difficult and require mathematical processing before interpretation. Therefore, it is necessary to clarify the coordinates of the carrier of the fuzzy membership function for each of them. The technique we have developed involves the construction of an experimental membership function for 2D electrophoresis, its approximation by a Gaussian or Bezier surface, and, then, determination of the fuzzy mass of proteins. The features of fuzzy protein recognition for 2D electrophoresis are considered.

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A Comparison of Monte Carlo Methods for Multidimensional Integrals in Air Pollution Modeling based on Latin Hypercube Sampling Edge Algorithm

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The present study is based on The Unified Danish Eulerian Model (UNI-DEM) as one of the most advanced large-scale mathematical models that describes adequately all physical and chemical processes. One of the most attractive features of UNI-DEM is its advanced chemical scheme the Condensed CBM IV, which consider a large number of chemical species and numerous reactions between them, of which the ozone is one of the most important pollutants for its central role in many practical applications of the results. The calculations are done in a large spatial domain, which covers completely the European region and the Mediterranean, for certain time period. A comprehensive experimental study of Monte Carlo algorithm based on modifications of the Latin Hypercube Sampling Edge and Random Algorithms for multidimensional numerical integration has been done. Samplings with different seeds has been analyzed. This comparison has been made for the first time for sensitivity analysis of UNI-DEM. The algorithms have been successfully applied to compute global Sobol sensitivity measures corresponding to the six chemical reactions rates and four different groups of pollutants.

Acknowledgements. The work is supported by the Bulgarian National Science Fund under Project DN 12-5 “Efficient Stochastic Methods and Algorithms for Large-Scale Computational Problems” and by the National Scientific Program “Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICT in SES),” contract No DO1-205/23.11.2018, financed by the Ministry of Education and Science in Bulgaria.

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A Parametric Investigation of the Effect of Maxwell-Cattaneo Heat Conduction on the Photothermal Radiometric Signal

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Photothermal wave techniques rely on the detection of subtle temperature changes of materials excited by intensity-modulated light. Due to their non-contact and non-destructive nature, they present several advantages over traditional material characterization methods. Here we focus on photothermal radiometry (PTR) which monitors the blackbody radiation emitted from a material that is optically excited by an intensity-modulated monochromatic light beam. Following partial or total absorption of the incident radiation, a portion of it is converted into heat. The subsequent diffusion of heat is dictated by thermal transport properties such as thermal diffusivity and thermal conductivity. PTR offers high sensitivity relative to the thermal and optoelectronic properties of materials, even though in most cases the extraction of the aforementioned parameters is not direct and involves significant computational effort.

In this work, we investigate how thermal relaxation affects the PTR signal. It was first argued by Maxwell that the classical Fourier heat conduction model violates the principle of causality since it predicts the propagation of heat at infinite speed. To address this, he amended the classical Fourier law by introducing a thermal relaxation time between the heat flux and the temperature gradient, which in turn leads to a damped wave equation for the temperature. Using this model, we conduct a parametric study to examine the impact of thermal relaxation time on the PTR signal.

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Stochastic Effects in the Eco-epidemiological Model with Multirhythmicity

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We consider the eco-epidemiological "predator-prey" model taking into account the Allee effect in the population of predators and dividing the prey population into infected and susceptible [1]. Depending on the Allee parameter, bifurcation analysis with the identification of zones of bi- and multirhythmicity is carried out. Spatial basins of attraction for equilibria and cycles are constructed. For the stochastic version of the model, probabilistic mechanisms of transitions between two different regular oscillatory regimes and between regular and chaotic ones are studied. The preferential modes that occur in the presence of random interference are determined. Using the stochastic sensitivity function and the method of confidence domains [2] we obtain estimates of the critical values of the noise intensity at which extinction occurs.

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Numerical Methods for Stochastic Sensitivity Analysis of 2D Chaotic Attractors

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In the theory of dynamical discrete-time systems, critical lines [1] play a key role. These lines allow one to study the dynamic properties of noninvertible maps and to describe the boundaries of a chaotic attractor. The previously constructed stochastic sensitivity of chaotic attractors [2] is based on critical lines and let us estimate the dispersion of random states around the chaotic attractor. However, the technical problem is complicated by the fact that the critical lines describe not only the external boundaries, but also structures inside the chaotic attractor. The paper presents constructive algorithms for finding the outer boundaries of chaotic attractors, based on a geometric selection of points of critical lines belonging only to the outer boundary. These algorithms are tested for complex non-convex forms of chaotic attractors. Based on these algorithms, we solve the problem of finding confidence domains around chaotic attractors of stochastic systems.

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Pure Cylindrical Bending of a Laminated Composite Plate Differently Resistant to Tension and Compression

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Composite materials are structurally complex materials consisting of two or more fractions with a clear boundary. It is assumed that the composite materials composed of reinforcing and matrix components. Due to their structure, composite materials have different moduli of elasticity in tension and compression. This work considers problem of pure cylindrical bending of a laminated composite plate. Each layer of plate is polymer-based unidirectional composite reinforced by thin carbon fibers. The fibers have high tensile stiffness and low stiffness upon compression. The bending equation are obtained using the variational Lagrange method and integral functional of potential energy. This equation is solved using the finite element method with Bell's triangular finite element. Numerical experiments are carried out, a comparison of solutions that consider and do not consider the difference in elasticity moduli is shown. The dependence of the deflection on the angle of reinforcement and the order of stacking layers in the composite material is shown.

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A Study on the Parameters of Manifold Mixup and Its Interaction with Other Regularization Techniques When Training Neural Networks

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Deep learning is currently state of the art at learning complex representations from training data. However, on test data neural networks can sometimes fail to predict correctly, while at the same time doing so with high confidence. One reason for this is the fact that after training, the latent representation of data might become non-linear, with very good ability of the model to predict the training examples, but with poor generalization to test examples. This is called overfitting, and to control for it different regularization techniques exist. They are usually employed during training to stop the neural network from learning the training data perfectly, thus modeling its inherent noise. One recently proposed and promising regularization method is manifold mixup [1]. The technique interpolates between the outputs of intermediate layers of the neural network on training time. This encourages it to build better representations, casting different classes of data further apart, while building broader, low-confidence margins between them. In the current study, the parameters of manifold mixup are systematically tested. One of these parameters is the mixing coefficient, which comes from a probability distribution (usually Beta) and specifies the level of interpolation between two training samples. Another parameter is the specific layer where interpolation takes place – beginning, middle or end layers. In addition, the interaction between manifold mixup and different commonly used regularization methods is also studied.

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Lie Group of the Second Degree Infinitesimal Conformal Transformations in a Symmetric Riemannian Space of the First Class

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The study of infinitesimal transformations in Riemannian spaces is of interest both theoretically and as an application. The distribution of relativistic gas according to the Maxwell-Boltzmann law is characterized by a vector $\xi(x)$, which is a Killing vector (if the gas consists of particles of nonzero rest mass) or an infinitely small conformal transformation vector (if the gas consists of particles of zero rest mass). Groups of conformal transformations have been studied to a much lesser extent than movement groups. The group of conformal transformations in spherically symmetric gravitational fields was investigated by Takeno, who indicated the complete system of solutions of the generalized Killing equations for the indicated spaces. A.Z. Petrov gave a classification of gravitational fields of general form according to the groups of infinitesimal motions and conformal transformations. A complete solution to the problem of classifying gravitational fields by groups of conformal transformations was obtained by R.F. Bilyalov, the main result of which is as follows: the group of conformal transformations acting in a non-conformally flat gravitational field is a group of motions or homotheties of a space conformal to a given one. In this article, infinitesimal motions in symmetric Riemannian spaces of the first class V_n were studied. For $n = 4$ the basis of the Lie group G_{12} of examined transformations is explicitly found and the structure of this group is given.

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On a Timoshenko Beam System with Dampling Boundary Conditions

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In recent years, operator analysis enetered in the classical beam theory as a powerful tool establishing both spectral properties and stability of whole classes of modelling equation systems. In the present work we show stability properties in the system establishing the semigroup type behind the scene and describing the important Fredholm operators and their spectrum sitting behind the abstract Timoshenko System.

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Nonlinear Acoustic Metamaterials: Modeling, Localized and Periodic Waves and Their Control

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The nonlinear modulation of waves in a metamaterial mass-in-mass lattice model is studied. This model accounts for an internal oscillator in a conventional 1D chain. An asymptotic procedure is developed to obtain the governing continuum nonlinear equation from the original discrete model. The features of the wave modulation in a metamaterial are studied on the basis of the exact and asymptotic solutions to the obtained model nonlinear equation. The differences in the modulation wave dynamics on the acoustic and optical bands are described analytically. A numerical procedure is developed to describe the formation of the waves in a metamaterial by a boundary periodic excitation. The band gap is revealed in an agreement with the analytical solution. It is shown how the switching on/off of the internal oscillator can control propagation of the waves in the metamaterial.

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Numerical Simulation of the Diffusion Instability of the Oregonator

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Studies of chemical, physico-chemical, and biological systems indicate the influence of component diffusion on the instability of processes. Diffusive instability is associated with the formation and development of spatial structures in systems of different nature. Stationary structures were observed not only in experimental studies, but also in working reactors. Objective: numerical simulation of the characteristics of unstable modes of the oregonator, taking into account the diffusion of components. For the five-stage Field-Keros-Noyes model of the Belousov-Zhabotinskii reaction, called the Oregonator, kinetic equations are presented without taking into account the inverse reactions, with the inclusion of the diffusion of components. To study the stationary state of the oregonator at different values of the stoichiometric coefficient, a computational algorithm is developed. The program for calculating the stationary states of the oregonator is written in Matlab. In computational experiments to determine the stationary states of the oregonator, the constants of the reaction rates obtained by the authors of the model are used. The transition to a system of partial differential equations for perturbations of component concentrations is carried out. The dispersion relation is derived. A computational algorithm and programs for calculating the characteristics of the oregonator under diffusion instability are developed. The results of computational experiments of unstable modes for different values of the stoichiometric coefficient, which is a bifurcation parameter of the system, are presented. Two types of unstable modes of the oregonator under conditions of diffusive instability, namely, the mode of oscillatory instability and the change of stability, are identified. The growth rates of perturbations in the system are calculated. The dependence of the perturbation growth rate on the value of the stoichiometric coefficient is shown. The regime of stability change is characterized by higher rates of perturbation growth in comparison with the regime of oscillatory instability.

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Study of Hydrodynamics and Mass Transfer Influence on Crystal Growth from Water-salt Solutions

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The regularities of spatial flows and mass transfer in supersaturated water-salt solutions during a number of crystal growth (potassium dihydrogen phosphate crystal –- KDP [1] and mixed nickel-cobalt crystal –- KCNSH [2]) has been studied. The solution flow occurs in a region of complex shape containing solid crystallizing bodies, the growth of which is determined by the conditions of their flow around (by the velocity and direction of flow around and the salt saturation and temperature of the solution). Two variants of crystallization are considered: in the first, the solution salt saturation is supplied using an external forced flow, and in the second, the solution salt saturation is supported by permanent solution cooling. Both cases are considered in laminar regimes at Reynolds numbers significantly lower than the critical ones. Moreover, the crystallizers sizes were increased and the flow around corresponded to the large Reynolds number and turbulent flow around. In this case, the turbulence models were used for numerical modeling. The crystal growth process is considered in a conjugate formulation as mass transfer in the "solution-crystal" system. It is shown, how local features of hydrodynamics and mass transfer near a growing crystal surface specifically affect on the local crystal growth rate and defect formation.

Acknowledgements. The present work was supported by the Ministry of Science and Higher Education within the framework of the Russian State Assignment under contract No. AAAA-A20-120011690136-2.

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Hierarchy of Integrable Equations and Lax Pair

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In this talk, we discuss hierarchy of integrable equations and Lax pair. I will present a two-component short pulse system produced through a negative integrable flow associated with the WKI hierarchy. The Lax representation will be given for the whole hierarchy. Particularly, some eigenvalue problems are provided for investigating the spectrum distribution. The multi-soliton solutions for the two short pulse system investigated, in particular, one-, two-, three-loop soliton, and breather soliton solutions are discussed in details with interesting dynamical interactions and shown through figures.

Acknowledgements. This is joint work with Qiaoyi Hu and Qilao Zha.

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Retrospective Review of the Bulgarian Insurance Market Using Time Series Analysis

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Insurance plays an important role in the economic development of any country. As a result of rapidly evolving technologies and the emergence of new industries, the corresponding response from the insurance business is required. Insurance covers a number of areas in various spheres of human life and everyday life. This implies a great variety and maintenance of modern standards, which are established over time. At the same time, as an economic unit, any crisis and financial fluctuations, as well as political changes have an impact on the insurance market. Bulgaria is a country with a hesitant financial structure, and this further complicates the development of the insurance practice. The object of study in the present work is the General Insurance Market in Bulgaria. The article aims to trace the pace of development of the insurance business in Bulgaria over the past fifteen years. According to available data from the official website of the Financial Supervision for Bulgaria, data on eighteen insurance products offered in Bulgaria have been reviewed. The analysis was conducted on the values of premiums and benefits paid to about twenty leading insurance companies. With the help of time series theory and in particular the Auto Regressive Integrated Moving Average (ARIMA) models, the aim is to identify trends and factors that influence the behavior of premium

income of insurance companies and the emergence of claims against them. The ARIMA models are properly for the current analysis because the insurance risks are characterized by trend component, seasonality and random fluctuations. Also, forecasts are presented. The comparative analysis shows which types of insurance have undergone the greatest development over the years and which of them are more resistant to changes in the economic environment. Comparison is made between the growth rate of the premiums and the claims. The idea is to highlight the prosperous industries and to notice the problem areas in the insurance business in our country.

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Risk Estimation Using Sensitivity Analysis of an Investment Project

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The essence of the sensitivity analysis of an investment project, consists in a comparative analysis of the influence of different factors on an investment's key indicator – its effectiveness. This analysis tries to assess the impact of changes in input data on the final characteristics of the project. A credible sensitivity analysis typically follows these steps: (1) Identifying the key indicator that measures the effectiveness of the investment; (2) Determining the factors which are in a state of uncertainty; (3) Establishing the nominal and limit (lower and upper) values of the uncertain factors, selected at the second step of the procedure; (4) Evaluating the key indicator for all selected limit values of the uncertain factors; (5) Plotting the sensitivity graph for all uncertain factors (Spider Graph). This method is a good illustration of the influence of individual factors on the success of the project. In the process of sensitivity analysis, the risk does not change immediately. Instead, the resilience of the project to changes in the parameters is determined. The higher the resistance to changes, the lower its risk. Thus, the project with the lowest NPV sensitivity to parameter changes is considered less risky. In the current paper is presented a method for risk evaluation by analyzing the sensitivity of an actual investment project. Also, the sensitivity of the project to the change in the discount rate E and the cash flows CFi is determined. Finally, the probability of NPV being less than a certain value K is examined.

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Solving *Pulsed Chemotherapy Model* of Fractional Order Using RKHS Method

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Over the last years, mathematical modeling has become a valuable tool for the dynamic analysis of infectious diseases and the support of control strategies development. Many mathematical modeling studies can be found in the literature such cancer. In our paper, we will concentrate on a cancer model. This latter is considered as an attractive field of research, since it is a fact that publications on mathematical modeling of cancer turn out to have quite an impact on our world nowadays. In this manuscript, mathematical model for cancer chemotherapy is proposed, taking into consideration normal-tumor cell interactions. Also, periodic chemotherapeutic treatments will be considered. The motivation behind this research was a published paper [1]. Thus, based on this discussion, the conformable definition will be applied on the model proposed in the previous paper. Doing so will offer insights about a new fractional version. Moreover, we will use Kernel Hilbert space method to solve the novel problem satisfying specific initial conditions in order to solve the novel problem. Applying this set of methods will help us to find approximate solution of the fractional novel model, noting that the solutions are presented in the Hilbert space. This method is carries a huge important scientific applications as far as numerical analysis, ordinary and fractional differential equations and learning theories are concerned. The precision offered by the adopted method requires less exertion to discover numerical results. This is why the RKHS method was successfully used by many authors to investigate several scientific applications side by side with their theories. Finally, to support our findings, we present several numerical simulations of the novel model to demonstrate the method's efficiency and accuracy. Then, we compare the obtained simulations with the previous results to show and highlight any significant difference that may occur.

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Fano Factor of Output Spike Trains for a Neuron Model Driven by Dichotomous Noise

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The Fano factor (FF), defined as the variance-to-mean ratio of numbers of events observed in a time window, is widely used in operational models to describe the stochastic repetition of a certain event. For instance, the areas investory control, traffic flow, communication networks, epidemic analysis, hydroelectric operations, information transfer within a neural system, etc., all use the stochastic theory of recurrent events. One of the most important open questions in neuroscience is how neurons code transferred information into spike trains. The neuronal spike train variability is usually examined based on renewal processes assuming independent and identically distributed lengths of inter-spike-intervals (ISIs) which certainly not always apply to real spike trains, e.g., often the ISIs are correlated. Although, the influence of the ISIs correlation on the limit value of the FF (i.e., in the limit if the observation window tends to infinity) is relatively thoroughly investigated, however, it is not clear how by finite length of the observation window the FF is affected with the ISIs correlation. Here we emphasize, that the knowledge of the behavior of FF in a finite observation window is crucial from the point of view of possible experiments. In this study, we describe and illustrate the dependency of FF on the length of the observation window and investigate the effect of subordination on this dependence. The spike trains considered possess ISIs correlation and are generated from two neuron models. Firstly, the output spike train of a perfect integrate-and-fire (PIF) model of neurons driven by asymmetric dichotomous noise is considered. Next, the model is extended by including a random operational time in the form of an inverse strictly increasing Lévy-type subordinator, and exact formulas for the FF are analytically explained also in this case. It is shown that the dependencies of Fano factors of both models on the observation window behave significantly different from renewal processes. Our main generic result is that the behavior of FFs in PIF models with subordination and without subordination are profoundly different. Particularly, it is shown that highly nonmonotonic dependence of the FF on the observation time (at small and moderate values of time) generated by the input dichotomous noise in the PIF model without subordination is removed due to memory effects in the model with subordination. Moreover, in the case with subordination the FF demonstrates at sufficiently large values of the observation time a nonmonotonic dependence on the mean input current μ, which indicates that the spike train regularity is maximized at this value of the input current. We believe that the obtained results not only supply material for theoretical investigations of more realistic neuronal models, but also suggest some possibilities for interpreting experimental data on neurobiological applications.

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A Data-driven Stochastic Framework for Treatment Assessment in Colon Cancer

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In this talk, we present a new stochastic framework for treatment assessment in colon cancer. The dynamics of colon cancer is given by a stochastic process that captures the inherent randomness in the system. The stochastic framework is based on the Fokker-Planck equation that represents the evolution of the probability density function corresponding to the stochastic process. An optimal control problem is formulated that takes input individual patient data with randomness present. The output is given as a set of optimal dosages that is used to assess efficacies of various treatments in colon cancer patients. Numerical experiments demonstrate the robustness and accuracy of our proposed framework.

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Mean Field Game Equations with Underlying Jump-diffusion Process

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We consider a couple of integro-differential PDEs arising from a stochastic marcovian control problem subjected by a initial-terminal conditions. These equations corresponds to the MFG system for a controlled jump-diffusion process with an exponential Poissonean kernel. We prove that for a specific choice of the control function the expectation of the jump-diffusion process can be found explicitly. The study is an extension of similar results known for the pure diffusion process. As an example, we show how it can be applied to a problem about an estimation of the trend of asset by investors during the optimal portfolio selection. The study is a continuation of similar results known for the pure diffusion process. As an example, we show how this can be applied to the problem of investors evaluating the trend of an asset when choosing an optimal portfolio.

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Numerical Simulation for the One Stationary Nonlinear Hydrodynamics Problem in Non-convex Domain

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The problem that is obtained using implicit time integration of a unsteady nonlinear incompressible Navier-Stokes equations in the rotation form in non-convex polygonal domain is considered. The weighted finite element method based on the concept of an R_ν-generalized solution is constructed. The advantage of the proposed approach over classical approximations is numerically established.

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On Body of Optimal Parameters in the Weighted Finite Element Method

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A weighted finite element method (WFEM) is constructed to find an approximate solution to the crack problem. We have shown that the reentrant corner 2π at the boundary of the domain does not affect the accuracy of finding the solution by this method. The approximate solution by the WFEM converges to the exact one with the rate of $\mathcal{O}(h)$. Three control parameters affect the accuracy of finding the approximate solution by the WFEM. In this paper we define the body of optimal parameters (BOP) in the WFEM for the crack problem. The error of the found approximate solution deviates from the smallest error by no more than a predetermined value when we choose parameters from the BOP.

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Mechanisms of Stochastic Analysis in the Individual-tree and Whole-stand Growth Models

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The most published studies on individual-tree and whole-stand growth methodologies have failed to incorporate structured stochastic mechanisms. Generally, they have relied on univariate deterministic logistic type models. In particular, stochastic structure of tree size components has been neglected. Methodologies for incorporating stochastic structure in growth models can be framed using diffusion processes. The Voronoi polygons were used to characterize spatial pattern of tree positions as well as the competition relations between the individual trees. From the mathematically point of view, the Voronoi polygons represents one of the best solutions to determine neighboring competitors of a tree. The dynamic of the bivariate probability density function of the tree size variables (available polygon area of a tree and tree diameter) was described by the mixed-effect parameters Gompertz type bivariate stochastic differential equation. Benefits from incorporation of stochastic structure include valid statistical inference, improved estimation efficiency, and more realistic and theoretically sound predictions and forecasts.

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Homogenized Problems with Convolutions for Diffusion and Filtration in Porous Media

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The investigation of the dynamic processes of diffusion and filtration of liquids in porous media is relevant when planning the use of underground resources and searching for ways to clean such resources from contamination. Studies of such processes by engineering observation methods are expensive and practically impossible. Therefore, modeling is the only way to predict and possibly optimize methods for cleaning and preventing contamination of underground resources. Porous periodic media, formed by a large number of “blocks” with low permeability, and separated by a connected system of “faults” with high permeability, will be considered here.

Taking into account the structure of such media in modeling leads to the dependence of the considered filtration equations and their coefficients on a small parameter characterizing the microscale of the porous medium and the permeability of the blocks. Thus, initial boundary value problems for nonstationary equations of filtration in such porous media are considered. Assertions about the solvability of such problems and the corresponding homogenized problems with convolutions are given. These statements are proved for general initial data and inhomogeneous initial conditions and are generalizations of classical results on the solvability of initial-boundary value problems for the heat equation. The proofs use the methods of a priori estimates and the well-known Agranovich-Vishik method, developed to study parabolic problems of general type.

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Modeling and Homogenizing Optimization Problem for Medical Microneedle Systems

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A mathematical model and homogenization method of parameter computation for transdermal (hypodermic) drug delivery by medical microneedle systems are presented. Such systems are formed by a large number of microneedles, which are fixed on plane base, and used for vaccine, protein and insulin injections in modern medicine. Numerous publications confirm the high efficiency of the microneedle system applications for transdermal (hypodermic) medicine injections at the treatment of different diseases. Microneedles of such systems, as a rule, are not ordinary medical needles. The microneedles are synthesized from biodegradable polymers that are dissolved with the prescribed rate after transdermal drug injections. The efficiency of using such systems depends significantly on the parameters of microneedles. The problem of determining such dependencies and optimal parameters will be considered as the problem of optimizing the interaction of microneedle systems with an elastic surface. Minimization problems for integral functional, whose solutions are approximations for solutions to the interaction problem, will be obtained by the homogenization theory methods. Such problems are formulated in the form of classical variational problems with obstacles. The obtained problems and values of microneedle parameters guarantee the effective and comfortable use of such systems for transdermal (hypodermic) medicine delivery. This is relevant and essential for

vaccine injections in the context of the current pandemic.

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Theoretical Study of the Stresses Change Nature Arising in the Polymer Concrete Compacted Layer When Compacted on a Vibrating Platform with Vertically Directed Vibrations

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Purpose. The purpose of these studies is the theoretical determination of the stress-strain state of the compacted medium of the dynamic system "vibration platform - polymer concrete."

Methodology. To theoretically determine the stress-strain state of the polymer concrete compacted by the movable frame of the vibrating platform, a study of the dynamic system "vibrating platform - polymer concrete" was performed, in which polymer concrete compacted by vibration loading is presented as a system with distributed parameters. During the theoretical research, the substitution of the expression describing the law of motion of the vibrating platform moving frame in the operating mode, in the dependence between stress and strain, which is written for the conditions of uniaxial stress.

Results. As a result, the law of change of stresses arising in the compacted layer of polymer concrete was determined. The analysis of the obtained expression allowed us to conclude that the amplitude of stresses arising in the polymer concrete layer is significantly influenced by the amplitude of forced oscillations of the moving frame of the vibrating platform, the angular frequency of forced oscillations, the height of the compacted polymer concrete layer; vibration load absorption coefficient.

Originality. New theoretical expressions are obtained, which allow to determine the change of stress amplitude in the base and on the surface of the polymer concrete layer, which is deformed by the movable frame of the vibrating platform. According to the obtained theoretical dependencies, graphs are constructed that clearly illustrate the peculiarities of changes in the amplitude of oscillations of the moving frame of the vibrating platform depending on the height of the product, stresses arising at the base and on the surface of the compacted polymer concrete layer at the selected vibration load. For the first time, graphical dependencies are constructed, which illustrate the peculiarities of stress change along the height of the compacted layer depending on the relative density of the polymer concrete layer for the selected vibration mode when modeling the compacted polymer concrete medium by Ziner's rheological model.

Practical value. Theoretical studies of the dynamic system "vibration platform - polymer concrete" allow to determine the modes of vibration exposure (amplitude and frequency of forced vibrations), design and technological features of vibration equipment.

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Enumeration of Some Families of Bargraphs

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In this presentation, we consider several statistics on some families of bargraphs. As main result, we will exhibit a bijection between the set of t-compositions and a set of bargraphs, named t-bargraphs. As the main statistics under consideration is per – the perimeter of a given t-bargraph, we obtain the corresponding generating function. We find both an explicit and an asymptotic formula for the total length of the perimeter over all t-bargraphs with n-cells.

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In-Advance and In-Time Managements in Mathematical Models of "Mean Field Games"

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In the talk, the mathematical economic models with the "Mean Field Game" structure is discussed for the predictive modeling under both in-advance given control conditions and in-time optimizing management to achieve the desired result. The mathematical model is a pair of parabolic partial differential equations (Fokker-Planck-Kolmogorov and Hamilton-Jacobi-Bellman types with a corresponding set of initial and boundary conditions) for minimizing a given cost functional. For this problem, the discretization is applied to obtain systems of nonlinear algebraic equations which are solved in an iterative way to get the best instant benefit for each agent. The authors present the special type of approximation inheriting the basic properties of a differential problem (conjugacy and monotonicity of operators) at a discrete level. This mathematical apparatus is used for the quantitative modeling of the distribution or the use of alternative resources, environmental problems, optimization of wages and insurance, network sales, and other socio-economic activities to predict the aggregate behavior of the great mass of agents looking for instant personal benefit.

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Structure and Functions of the Intelligent Software Complex for Modeling the Solution of Optimization Problems of the Bank's Retail Block

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This report describes the structure and functions of the modules of an intelligent computer software complex designed for modeling and solving of optimal program and adaptive control of the number of employees and the sales system of the Bank's Retail Block. To solve these problems, the authors have developed a discrete-time controlled dynamical economical and mathematical model of the investigated business processes, which served as the basis for creating a software package. The report presents the main stages of creating the proposed controlled dynamical model in the presence of a vector criterion for the quality of realization of the considered banking processes. Within the framework of this model and the formalization of the considered problems of vector optimization of program and adaptive control of the processes under study, the corresponding numerical algorithms have been developed, which are implemented as stand-alone modules in the Delphi software environment. The report presents the results of computer simulation of the solution of various options for practical examples, for which the desired optimal

solutions were obtained using the developed intelligent computer software package. There is also a graphical illustration of the results obtained and their analysis. The proposed controlled dynamical model also allows solving other problems of optimizing program and adaptive control of processes that determine the activities of financial organizations and developing appropriate automated information systems to implement support for making managerial decisions.

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Stability of Notched Cylindrical Shells under Uneven External Pressure

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The goal of the paper is an investigation of a bearing capacity of a notched cylindrical shell unevenly loaded with wind-type pressure. The assumptions made in determining the subcritical stress-strain state of the construction are essential. The large notches presence leads to the appearance of deflections, which commensurate with the shell thickness even at low load levels. Consequently, the linear formulation does not allow revealing the true nature of the buckling process and can lead to incorrect results. The proposed calculation scheme is adopted for the purpose of rational placement of hatches, portholes and automation units. It can be used to study the "survivability" of the construction in case of sudden damage. A variational problem statement is used. The unknown functions are approximated in the meridional direction using cubic Hermite interpolation polynomials ensuring consistency. After substituting the approximations into the functional and drawing up the Euler equations for unknown functions, we obtain a system of differential equations. The numerical algorithm is based on reducing the boundary value problem to the Cauchy problem, and applying the Runge-Kutta method. The solution continuation with respect to the load parameter is used with the parameter change at the singular point. New results are obtained for calculating notched shells under non-axisymmetric pressure when the subcritical stress-strain state is determined taking into account geometric nonlinearity. Stability loss occurs by the type of reaching the limiting point. The critical pressure significantly depends on the relative position of the notch zone and the maximum of the pressure diagram. It is shown that the ultimate loads increase with the distance of this maximum from the notch zone. The minimum critical load under non-axisymmetric pressure coincides with the ultimate load under uniform pressure. There is a significant difference in the non-uniformity effect on critical loads for smooth and notched shells.

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Noise-induced Bursting in the Region of Tonic Spiking Behavior in the Neuron Model with Lukyanov-Shilnikov Bifurcation

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We consider a problem of mathematical modeling and analysis of stochastic phenomena related to complex oscillatory regimes in neural activity. We study the three-dimensional Hindmarsh-Rose model with Lukyanov-Shilnikov bifurcation. This bifurcation is characterized by the existence of the parameter region of bistability where the system exhibits a coexistence of two limit cycles representing tonic spiking and bursting. Here, we consider the parameter zone where the only attractor is a tonic spiking limit cycle and show that noise can generate bursting oscillations in this region. This stochastic phenomenon is confirmed by changes in the probability density distributions for phase trajectories as well as by temporal characteristics of oscillations. We obtain statistics of changes in interspike and interburst intervals, as well as distributions of frequencies of oscillations in dependence on the noise intensity. Increasing mean values of interspike intervals indicate an appearance of long phases of quiescence observed in the bursting regime, while histograms of distribution of frequencies show a transition from uni- to bimodal form.

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Numerical Solution of Equations Describing the Action of Electric Field on a Liquid Crystal

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To study dynamic processes in liquid crystals a general mathematical model describing mechanical, temperature and electric perturbations was proposed in [1]. Based on the equations of the model a subsystem of second-order equations for the tangential stress and angular velocity was obtained and investigated in [2]. This subsystem describes only mechanical impact. Computational algorithm describing the electric field in a liquid-crystal layer is proposed in [3]. In current work, the subsystem of two equations is generalized to take into account the electrical perturbations. An extended liquid-crystal layer is placed between the plates of a capacitor. The electric field arises due to the appearance of charges on the plates. The first stage of numerical algorithm is to calculate the electric potential using the method of straight lines and iterative method. Then the tangential stress and angular velocity are calculated by means of a finite difference scheme "cross." Based on the Fourier spectral analysis, the stability of the scheme is shown. Computational algorithm is implemented as a parallel program written in the C++ language using CUDA technology for computer systems with graphics accelerators. To demonstrate the operability of the algorithm and the program a series of calculations was performed for the reorientation of liquid crystal molecules under the electric field action.

Acknowledgements. This work is supported by the Krasnoyarsk Mathematical Center and financed by the Ministry of Science and Higher Education of the Russian Federation in the framework of the establishment and development of regional Centers for Mathematics Research and Education (Agreement No. 075-02-2020-1631).

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Data Transformations and Transfer Functions

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During the talk we speak on learning mechanisms of data transformation and aggregation. This will be connected to the information theory approach to Machine learning, Neural Computing and Artificial Intelligence, which may provide us with a new perspectives of methodological and applied research for statistical inference and optimal design. Several questions will be addressed, e.g. What is optimal learning of complex data? This will be also addressing advanced transfer functions for neural networks, as SPOCU. Can we learn negatively? Counterexamples, and several paradoxes on regular and singular models will be provided.

Importance of Statistics and Optimal designs will be acknowledge in a ecumenic way to Machine learning and neural networks. Some advanced statistical techniques, like from algebraic statistics and nonparametric statistics will be advertised. I will also introduce topological and semi-topological data analysis (TDA and STDA) for evolving complex data systems. Applications to image analysis, finance, ecology, machine learning, neural networks, neural computing and neuroscience will be given.

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Symmetric Diophantine Systems

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We will consider applications of the local-to-global Hase principle for symmetric Diophantine equations and systems. Natural examples of symmetrical Diophantine systems arise in Euclidean geometry, in problems for integer lengths of elements of geometric figures. The questions of validity of the Hase principle and parametric description of infinite series of nontrivial solutions are covered.

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Oscillatory Flow of Carreau-Yasuda Fluid in a Pipe

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Some viscoelastic polymer fluids have very strange behavior due to the strong non-linearities of their mechanical properties. The study of such flows in pipes is of great interest for different practical purposes, such as 3D printing. Although the Reynolds number may be sufficiently low to assume laminar flow regime, i.e., parabolic flow, at some high pressure gradients the flow becomes unstable. In this work, we study theoretically what are the combinations of parameters to preserve the parabolic flow type. The viscosity is a non-linear function of shear rate and is modeled by the Carreau-Yasuda model. Concerning an oscillatory pressure gradient, the flow problem is reduced to a non-linear PDE of parabolic type for the axial velocity. For some values of the parameters and shear rates on the pipe walls, the equation is forward parabolic, while at others – backward parabolic, which does not possess any generalized solution. It is proven that the velocity and its gradient (shear rate) are bounded by constants, which depend on the Carreau number (or Weissenberg number) and the other parameters of the viscosity model.

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Regularization Methods of conic Optimization

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We study new ways of regularization of conic problems, which consists of transforming a problem to an equivalent form, where the Slater condition is satisfied and, therefore, the strong duality holds. Our approach is based on the concept of immobile indices which plays an important role in the feasible cone representation.

For the case of linear copositive problems, we obtain new representations of the minimal faces containing its feasible set and describe a regularization algorithm based on the obtained representations.

This algorithm is compared to other regularization procedures developed for a more general case of convex conic problems and based on a facial reduction approach.

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Hierarchical Game with Random Second Player and Its Application

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A hierarchical game of two players with a random second player is considered. Optimal strategies are defined on the base of Stackelberg equilibrium. The random second player is understood as a randomly selected person from a homogeneous set of decision-makers. The model can be used in various problems. First of all, it is may be used for optimal price choosing for a new product. For example, model is applied to the problem of setting the optimal fare. A carrier acts as a first player, a randomly selected passenger acts as a second player; it is assumed that the function of his preferences depends on a random parameter. A model example is considered.

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Calculation of the Optimal Timeout of Executing a SQL Query for Multi-Master Replication System

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Many multi-master replication systems are based on a two-phase commit mechanism. One of its problems is waiting for a response from the database servers by the coordinator, which may not come due to problems on the network or on the database server itself, which leads to delays in the activity of the multi-master replication system. Usually, such problems are solved by introducing a timeout, but if its value is too large, the problem may not be noticed in a timely manner, and too small a timeout will be processed for SQL queries, which will cause an increase in the load on the network due to additional service messages. The paper analyzes the execution time of a SQL query on several servers in order to calculate the most optimal timeout for a multi-master replication operation. Based on the results obtained, a module for calculating the optimal query execution timeout for multi-master replication systems has been developed.

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Highly Efficient Stochastic Methods for Sensitivity Study of Large-scale Air Pollution Model

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We discuss a systematic approach for sensitivity analysis studies in the area of air pollution modeling. Different parts of the large amount of output data, produced by Unified Danish Eulerian model, were used in various practical applications, where the reliability of this data should be properly estimated. Another reason to choose this model as a case study here is its sophisticated chemical scheme, where all relevant chemical processes in the atmosphere are accurately represented. We study the sensitivity of concentration variations of some of the most dangerous air pollutants with respect to the anthropogenic emissions levels and with respect to some chemical reactions rates. Quasi-Monte Carlo algorithms based on Faure, van der Corput and lattice sequences are used in our sensitivity study of the Unified Danish Eulerian model. A highly efficient lattice reflection rule is developed and applied to the model for the first time. The numerical tests will show that the stochastic algorithms under consideration, and especially the new lattice reflection rule, are efficient for the multidimensional integrals under consideration and especially for computing small by value sensitivity indices. Clearly, the progress in the area of air pollution modeling, is closely connected with the progress in reliable algorithms for multidimensional integration.

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Assessment of Temperature Uniformity and Optimization of the Heat Treat Furnace Working Area

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The development of metallic materials for aerospace applications requires a controlled process of the heat treatment in order to increase certain mechanical properties, such as metal hardness or strength, to improve ductility, wear resistance, corrosion resistance, toughness, and impact resistance. The heat treatment processes require the precise control of temperature over the heating cycle. Laboratory furnace needs to be calibrated in order to meet the measurement traceability according to the Aerospace Material Specification (R) Pyrometry Rationale SAE AMS2750 REV. E. The objective of this work is to assess the temperature uniformity of the heat furnace according to "AMS 2750E Heat Treatment Standard and Calibration" requirements and to use a Matlab virtual model to obtain the temperature map and based on this to optimize the heat furnace working area. The paper describes the methodology used for temperature uniformity assessment and optimization of the Nabertherm LH 30/14 Model 238237 furnace working area. The experimental method involved two case studies, one for 870ϵC (case study 1) and one at 1000ϵC (case study 2). The Nabertherm furnace is used for heat treatments of parts made through additive manufacturing process using INCONEL 625. In order to obtain accurate results of actual furnace capabilities, 15 measurements location were established (sensors arrangement) and a 3D linear interpolation of the obtained results were computed to determine the optimum working area of the furnace.
 The furnace class uniformity ranges are defined in AMS 2750E and the novelty of the present paper is in the method used to establish a working area inside the furnace that meets requirements for a superior furnace class, required by the heat treatment of the materials used for aerospace applications.

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Discrete Rogue Waves in Complex Cubic Volterra Equation

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We study discrete rogue waves in a complex cubic Volterra equation. We numerically solve complex cubic Volterra equation under periodic boundary conditions. If an appropriate initial condition is provided, discrete rogue waves occur. We show that their amplitudes gradually increase and are periodic in the propagation direction.

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SQPDFO – a Trust-region Based Algorithm for Generally-constrained Derivative-free Optimization

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Derivative-free optimization is a specific branch of mathematical optimization where first and higher order derivatives of the objective function of the optimization problem are not available, too expensive to compute or too inexact to be used. Such problems do arise in many application areas, e.g., in engineering design optimization, wastewater treatment and quantum chemical processes. As only function value information and no derivative information is available, SQPDFO applies different sampling techniques to build local interpolation models of the objective and constraint functions and uses a self-correcting error technique (Scheinberger, Toint, 2010) which guarantees the quality of these models and their derivatives during the optimization process. Throughout the optimization process, first and second order derivatives of these models are used. SQPDFO can handle nonlinear and linear equality and inequality constraints and simple bounds on the variables. It is based on the SQP (Sequential Quadratic Programming) method which solves a sequence of optimization subproblems, each of which optimizes a quadratic program of the original optimization problem. Inequality constraints are carefully handled by slack variables which are not included in the local models to not unnecessarily increase the size of the interpolation matrix. We will present numerical results on a large set of academic test problems from the well-known optimization library CUTEst showing the good performance of the implementation of SQPDFO. Furthermore, we extended the code to run in parallel and several function evaluations can be used in each iteration. This was especially useful when applying SQPDFO to a very big multidisciplinary DLR research shape design problem of an entire airplane. As one complete function evaluation of the top-level optimization problem takes 56 hours, a code which needs a minimum number of iterations is crucial. We will show how SQPDFO is able to find a good solution within a very small number of iterations.

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Optimization for the Delivery of Parcels

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The problem of constructing an optimal complex route for the parallel delivery of parcels, for example, using drones, is considered. An algorithm for parallel delivery of parcels has been developed, taking into account the energy efficiency of vehicles and drones, depending on the weights of the parcels. It also minimizes the carbon footprint or environmental damage and energy costs.

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Multiscale Model Reduction for Neutron Transport Problems in SP3 Approximation

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The SP3 approximation of the neutron transport equation allows improving the accuracy for both static and transient simulations for reactor core analysis compared with the neutron diffusion theory. Besides, the SP3 calculation costs are much less than higher order transport methods (SN or PN). Another advantage of the SP3 approximation is a similar structure of equations that is used in the diffusion method. Therefore, there is no difficulty to implement the SP3 solution option to the multi-group neutron diffusion codes. We attempt to employ a model reduction technique based on the multiscale method for neutron transport equation in SP3 approximation. A Generalized Multiscale Finite Element Method (GMsFEM) is used for coarse-grid discretization. Multiscale basis functions that incorporate small scale heterogeneities into the basis functions are constructed. The application of the SP3 methodology based on solution of the Lambda-spectral problems has been tested for the some reactor benchmarks. To verify the obtained results, we calculate relative errors between the multiscale and reference (fine-grid) solutions for different numbers of multiscale basis functions.

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Statistical Study of Particulate Matter (PM10) Air Contamination in the City of Vidin, Bulgaria

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It is well known that the high levels of all air pollutants are of great importance for human health. Especially dangerous are particulate matter (PM) contaminants. PM is a mixture of solid particles and liquid droplets found in the air. Particle pollution includes: PM10 inhalable particles with diameters that are between 2.5 and 10 micrometres and PM2.5 fine particles with diameters that are generally 2.5 micrometres and smaller. These particles are emitted directly from a source, such as construction sites, unpaved roads, fields, smokestacks or fires. Most particles form in the atmosphere as a result of complex reactions of chemicals such as sulfur dioxide and nitrogen oxides, which are pollutants emitted from power plants, industries and automobiles. For all European countries there are many regulations aimed against the air contamination and for the monitoring of pollution. Bulgaria, as a part of EU has to follow all these regulations. Still in Bulgaria there are many places and periods of time during the years with PM10 contamination, bigger than the daily norm of $50\mu g/m^3$. This study is to investigate the PM10 air pollution in the city of Vidin, Bulgaria for the period 2010–2020. The town of Vidin is located in north-western Bulgaria, on the banks of the river Danube – the north Bulgarian border with Romania. For the study we use official data from the monitoring of PM10 in Vidin region, Bulgaria. We apply different statistical methods to study data and to predict future PM10 pollution in the city of Vidin. All result of study are graphically presented and commented.

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Time Series Analysis of the Spread of COVID-19 Infection in Bulgaria

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The paper examines the dynamic in the deaths from COVID-19 in Bulgaria since the beginning of the pandemic in the region. Different time series approaches were used for modelling daily and weekly data separately and the results were compared. After smoothing the data, point forecasts together with confidence intervals were prepared. The results were visualised and the predictions were compared with the observed data. The considered models showed good prediction for the next 14 days. The obtained results can be used for simulation and forecasting of future values in order to prepare the right management decisions.

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A Symplectic Fourth-order Accurate Numerical Method for the Double Dispersion Equation

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In this paper the one-dimensional double dispersion equation is considered. The numerical method used for its solution is based on the representation of the equation as a Hamiltonian system. Replacing the space derivatives with finite differences with fourth order of approximation we derive a semi-discrete finite-dimensional system which is also a Hamiltonian system. A symplectic partitioned Runge-Kutta method with 3-stage Lobatto IIIA and IIIB coefficients is applied

for the time discretization. The developed numerical method is symplectic, i.e., its solution preserves the symplectic structure on the discrete level. Numerical experiments are provided for two specific problems: propagation of a single solitary wave and interaction of two waves traveling toward each other. The numerical results show $O(h^4 + \tau^4)$ order of convergence of the discrete solution to the exact one and conservation of the discrete energy, mass and momentum with a high accuracy.

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Conservative Semi-Lagrangian Approximation for Three-dimensional Parabolic Equation

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We present the semi-Lagrangian approximation of transfer operator for three-dimensional parabolic equation with corresponding initial and boundary conditions. This problem describes, for instance, a transfer of a substance with diffusion. We determine the numerical solution as a trilinear function at each time level on uniform time and space grids. To construct numerical method, we decompose operator of parabolic equation into two parts. The first one is the transfer operator. The second part is the elliptic diffusion terms. To approximate the first part, we use conservative semi-Lagrangian approximation where we get two integrals of solution at neighboring time levels. We approximate the integral domains and the solution to compute these integral analytically. The second part is approximated by finite differences. Finally, we combine approximations of two parts to get a system of linear equations. We find out a restriction for ratio between time and space grid steps to provide non-positive off-diagonal elements of system and diagonal dominance in columns which give the property of M-matrix at each time level. The proposed difference scheme has the first-order convergence that is confirmed by computational experiments.

Acknowledgements. The reported study was funded by Russian Foundation for Basic Research, project 20-01-00090 “Euler-Lagrangian (semi-Lagrangian) methods of finite differences and finite elements with special properties.”

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Global Well-posedness of Coupled Parabolic Systems for Two-component Combustion

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The initial boundary value problem of a class of reaction-diffusion systems (coupled parabolic systems) with nonlinear coupled source terms is considered in order to classify the initial data for the global existence, finite time blowup and longtime decay of the solution for two-component combustion. The whole study is conducted by considering three cases according to initial energy: low initial energy case, critical initial energy case and high initial energy case. For the low initial energy case and critical initial energy case the sufficient initial conditions of global existence, long time decay and finite time blowup are given to show a sharp-like condition. And for the high initial energy case the possibility of both global existence and finite time blowup is proved first, and then some sufficient initial conditions of finite time blowup and global existence are obtained respectively.

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Computational Method for Solving Differential Equations in Dynamic Systems

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In this article, we propose a computational method for solving a differential equation that are used for dynamic measurements in automatic control systems. These equations describe the state of a dynamic system at a real time. The proposed computational scheme of the method is based on finite-difference analogs of partial derivatives. The problem of the stability of the method solution high-order differential equations is one of the central problems of data processing in automatic control systems. The study of the stability of the method for solving differential equations in dynamical systems is relevant. The main goal of the computational experiment was to construct a numerical solution to the problem under consideration. Standard test functions were considered as input signals. The article studied the stability and convergence of the proposed method. In the course of the experiment, an estimate was found for evaluating the accuracy of the method.

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Identification of an Unknown Source Term in a Heat Conduction Problem from Boundary Measured Data

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In this work, we consider the inverse source problem related with heat transfer in a inhomogeneous linear object when the temperatures at the both its end are changed in time and available for measuring. This problem arises in non-destructive testing for building, structure and material inspections. The mathematical model of heat transfer is represented as one-dimension heat conduction problem for a parabolic PDE with unknown time-depend source term. In this problem, the source function must be recovered from the given boundary conditions that are formed from temperature measurements. Then the thermal fields at the internal points of the object must be calculated. To solve inverse source problem, the initial problem is reduced to an integral equation of the first kind describing the explicit dependence of the source function on the given boundary function. Then the computational method for determining the source function from integral equation is proposed. The applicability and efficiency of method is evaluated by comparing the numerical solutions of integral equation with the test functions. The computational results illustrate the sufficient reliability of the proposed algorithm.

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Field Theory of Deformation and Fracture

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A field theory that describes the deformation and fracture of solids is discussed. Previously we discussed that the present theory describes all stages of deformation including the fracturing stage comprehensively. A recent analysis indicates that this theory is applicable to fatigue of solids as well. Numerical models have been built to simulate fatigue tests. The results of the numerical analysis show qualitative agreement with experiments.

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Modeling the Optimal Trajectories for the Movement of Objects with Variable Structure in a Viscous Medium Density Nonconstant

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The formulation of the problem of optimal movement of a rigid body with variable structure from one phase state to another in a viscous medium with variable density is considered. The analysis of nonlinear relationships of the physical characteristics of a viscous medium is carried out and a mathematical model is built taking into account these relationships. The features of the problem of optimal control of the motion of bodies of variable geometry in a viscous medium of variable density from the point of view of the theory of optimal control are revealed.

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